Application of First order ODE

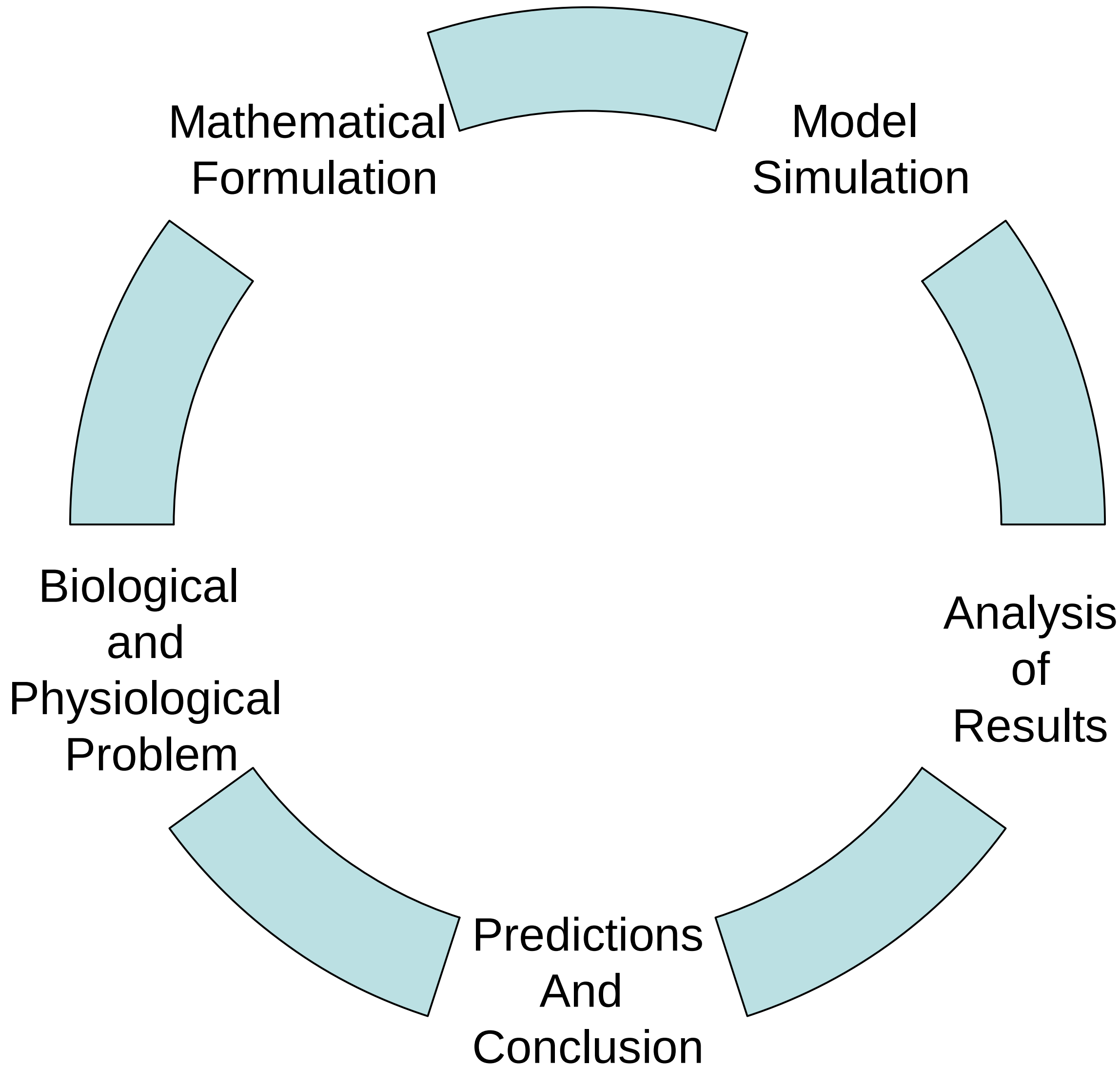
Mathematical Modelling

- The main reason for solving many differential equations is to try to learn something about an underlying physical process that the equation is believed to model.
- In the modelling process, we are concerned with more than just solving a particular problem. The complete solution process consists of the following steps.

- In the application of DE’s, we are concerned with more than just solving a particular IVP.

- The complete solution process consists of the following three steps:

- Construction of a mathematical equation and its initial conditions
- Solution of the mathematical equation's
- Interpretation of the results


Mathematical Formulation
Model Simulation
Analysis of Results
Predictions And Conclusion
Biological and Physiological Problem

Applications to Biology

- Biology is a natural science that deals with the study of living things and their interactions with their environment.

- In their study, biologists make use of mathematical models containing differential equations which enable them come out with laws regarding the behaviour of living things in relation to their environment

The simple linear differential equation

$$\frac{dy}{dt} = ky$$

arises in numerous physical theories concerning either growth or decay of some entity

Example 6.1 *(Radioactive decay problem)*

Experimental evidence indicates that a radioactive substance decays at a rate directly proportional to the amount present. Starting at time $t = 0$ with y_0 grams of un-decayed matter, find the amount present at some later time.

Example 6.2 *(Population and growth problem)*

The population of a particular community is observed to increase at a rate proportional to the number of people present at any one time. In the last five years the population has doubled. How many years will it take for the population to triple?

Applications to Economics

- Suppose we have a commodity such as maize or palm oil. Let p be the price of this commodity for some specified unit (e.g., a bag of maize or a gallon of palm oil) at any time t.

- Then we can think of p as a function of time t so that p(t) is the price at time t.

- The number of units of the commodity which is desired per unit time by consumers at any time is called the demand and is denoted by D(t), or briefly D:

$$D(t) = f(p(t), p'(t))$$

- Similarly, the number of unit of the commodity, which is made available per unit of time by producers at any time t is called the supply and is denoted by S(t), or briefly S:

$$S = g(p(t, p'(t))$$

Economic Principle of Supply and Demand

The price of a commodity at any time t is determined by the condition that the demand at t is equal to the supply at time t. Thus if

$$\left.\begin{aligned} D &= f(p(t), p'(t)) = a_1 p(t) + a_2 p'(t) + a_3 \\ S &= g(p(t), p'(t)) = b_1 p(t) + b_2 p'(t) + b_3 \end{aligned}\right\}$$

Then

$$a_1 p(t) + a_2 p'(t) + a_3 = b_1 p(t) + b_2 p'(t) + b_3$$

Example 6.3

The demand and supply of a certain commodity are given in thousands of units respectively by. If at the price of the commodity is 10 units, find

(a) the price at any later time .

(b) whether there is price stability or instability.

Applications to Chemistry

- Chemistry is the scientific study of the structure of substances, how they react when combined or in contact with one another and how they behave under different conditions.
- Mathematical models are used in chemistry to solve problems of a chemical nature.

Law of mass action

This law says that the rate of a chemical reaction is proportional to the product of the concentrations of the reactants.

- The rate at which a substance is formed in a chemical reaction is referred to as the speed of the reaction.

- In chemistry emphasis is on producing cost effective substances in the fastest possible way. Thus knowledge of the reaction rate or the speed of the reaction can be related to the concentration of the reactions and would be of immense importance

Example 6.4

In a bimolecular reaction, two substances are present. Suppose one molecule of A combine with one molecule of B to form one molecule of C. This chemical reaction is written as

$$A + B \rightarrow C$$

Let c be the molecule of C formed at time t and let a and b be the original numbers of molecules of *A* and *B* respectively.

Since a molecule of combines with a molecule of to form a molecule of , the number of molecules of and present at time *t* are respectively a-c and b-c.

From the law of mass action, a differential equation for this reaction is given by

$$\frac{dc}{dt} = k(a - c)(b - c)$$

where *k* is a constant of proportionality.

Applications to Medicine

- An important problem of biology and medicine deals with the occurrence, spreading, and control of a contagious disease, i.e. one that can be transmitted from one individual to another.
- The science that seeks to study such problems is called *epidemiology*, and if an unusually large percentage of a population gets the disease, we say that there is an *epidemic.*

- let us assume that we have a large but a finite population.

- To fix our ideas, let us suppose that we restrict ourselves to students in UEW who remain on campus for a relatively long period of time and do not have access to other communities.

We shall suppose there are only two types of students,

- those who have the contagious disease, called *infected*, and
- those who do not have the disease, i.e., *uninfected*, but who are capable of developing it on first making contact with an infected student

Suppose that at any time t there are N_i infected students and N_u uninfected students. Then if N is the total number of students, assumed constant, we have

$$N = N_i + N_u$$

The time rate of change in the number of infected students is then given by the derivative dN_i/dt. Assuming that dN_i/dt is a quadratic function of N_i as an approximation we have

$$\frac{dN_i}{dt} = a_0 + a_1 N_i + a_2 N_i^2$$

We expect dN_i/dt to be zero where $N_i = 0$, i.e., there are no infected students, and where $N_i = N$, i.e., all students are infected. Then we have

$$a_0 = 0 \qquad a_1 N + a_2 N^2 = 0 \qquad a_2 = -\frac{a_1}{N}$$

So we get

$$\frac{dN_i}{dt} = \frac{a_1}{N} N_i (N - N_i)$$

or

$$\frac{dN_i}{dt} = kN_i(N - N_i)$$

where

$$k = a_1 / N$$