B.Sc.(Hons.) Mathematics – III year
Test
Analysis – IV

Time: 1Hr. Max. Marks: 25

(This question paper consists of 6 questions. All questions carry 5 marks each.)
(Attempt any 5 questions.)

1. By changing the order of integration, prove that

$$\int_0^1 dx \int_x^{\frac{1}{x}} \frac{y^2\, dy}{(x+y)^2 \sqrt{1+y^2}} = \frac{2\sqrt{2}-1}{2}$$

2. State Green's Theorem and verify it for

$$I = \int_C \left[\left(x + y^2 \right) dx + \left(x^2 - y \right) dy \right]$$

Where C is the closed curve formed by $y^3 = x^2$ and $y = x$ between (0, 0) and (1, 1).

3. Use the transformations $u = \frac{x^2+y^2}{x}$, $v = \frac{x^2+y^2}{y}$ to evaluate $\iint_E xy\, dx\, dy$ over the region common to the circles $x^2 + y^2 = x$, $x^2 + y^2 = y$.

4. Evaluate $\iint_E x^{m-1} y^{n-1} (1-x-y)^{p-1}\, dx\, dy$; $m \geq 1, n \geq 1, p \geq 1$

where E is the region bounded by $x = 0$, $y = 0$, $x + y = 1$.

5. Evaluate:

$$\int_0^{2a} dx \int_{\sqrt{2ax-x^2}}^{\sqrt{4ax-x^2}} \left(1 + \frac{y^2}{x^2} \right) dy$$

by using $x = r\,\text{Cos}^2\theta$, $y = r\,\text{Sin}\theta\,\text{Cos}\theta$.

6. Evaluate:

$$\iint (x^2 + y^2)\, dx\, dy,$$

over the region bounded by $xy = 1$, $y = 0$, $y = x$ and $x = 2$.