ELITE PROJECT – MATHEMATICAL SKETCH OF SYMMETRIC GROUP OF DEGREE 5 (S₅)

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ABSTRACT

In this project, we worked on the Symmetric Group S₅, did its mathematical analysis and made the sketch of S₅ using its normal rotation. considering one of 5-cycle of S₅ we get 5 different cycles using its rotation. Again we have 5!/5 =24 choices of different cycles and hence 120 distinct 5- cycles, in this manner we plot these 120 elements and get a symmetric figure in 2 –dimensional plane whose points are interconnected in 3 –dimensional space. In the similar manner we plot the points of A₅ and get a structure is similar to Ashoka Chakra which is the part of National Emblem.
INTRODUCTION

Symmetric group is the set of all bijective functions from a finite set to itself, and forms a group under function composition.

Let $S = \{1,2,3,4,5\}$, then

$S_5 = \{f \mid f: S \to S \mid f \text{ is bijective}\}$, is a symmetric group under function composition.

CHARACTERISTICS

- Order of $S_n$ is $n!$.
- $|S_5| = 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$.
- Maximum order of any element in $S_5$ is 6.
- $S_5$ is a group of even order and has 60 even permutations and 60 odd permutations.
- $S_5$ is non-abelian.
- $S_5$ has 156 subgroups.
- Set of all even permutations is a subgroup of $S_5$ called alternating group of degree 5.
- $S_5$ has 3 normal subgroups, which are $\{e\}$, $A_5$ and $S_5$.
- $S_5$ is non-simple group as it has one non-trivial normal subgroup $A_5$.
- $Z(S_5) = \{e\}$. 
TABLE FOR ELEMENTS OF $S_5^{[1]}$

No. of the permutations (say $p$) based on the cyclic decomposition of a permutation in $S_n$ then,

$$p = \frac{n!}{1^{k_1} 2^{k_2} \cdots n^{k_n} K_1! K_2! \cdots K_n!}$$

Where 1, 2, 3, ...., n is the length of disjoint cycles occurs in the decomposition and $K_i$‘s denotes the occurrence of i-cycle.

<table>
<thead>
<tr>
<th>Possible cyclic decomposition</th>
<th>Order of element</th>
<th>No. of such Permutation</th>
<th>Type of element</th>
</tr>
</thead>
<tbody>
<tr>
<td>{1,1,1,1,1}</td>
<td>1</td>
<td>1</td>
<td>I={e}</td>
</tr>
<tr>
<td>{1,1,1,2}</td>
<td>2</td>
<td>10</td>
<td>(a b)</td>
</tr>
<tr>
<td>{1,2,2}</td>
<td>2</td>
<td>15</td>
<td>(a b)(c d)</td>
</tr>
<tr>
<td>{1,1,3}</td>
<td>3</td>
<td>20</td>
<td>(a b c)</td>
</tr>
<tr>
<td>{2,3}</td>
<td>6</td>
<td>20</td>
<td>(a b)(c d e)</td>
</tr>
<tr>
<td>{1,4}</td>
<td>4</td>
<td>30</td>
<td>(a b c d)</td>
</tr>
<tr>
<td>{5}</td>
<td>5</td>
<td>24</td>
<td>(a b c d e)</td>
</tr>
</tbody>
</table>
FIGURE OF $S_5$
STRUCTURE OF $S_5$

- **Deflated football**: According to the structure made by us we could easily observe that figure of $S_5$ looks like deflated football in 2-dimensional plane.

- Points at the corner are connected to the points on other side. This concludes that it is a 3-dimensional structure.

- As in the structure of football, pentagons are connected with hexagons but in our structure every pentagon is connected with a square or rhombus. This makes structure of $S_5$ different from football.

- **Starfish**: Outer boundary figure looks like a Starfish. A Starfish has five legs. There is an angle of $72^\circ$ between two consecutive legs which gives us a structure like symmetry of pentagons.
Chemists will definitely say that it is the structure of their well-known carbonic compound called “fullerene”. Fullerene is a well-known and very famous carbonic compound it is often called $C_{60}$.

![Structure of Fullerene](image1)

**Structure of IF$_7$ (Iodine Hepta Fluoride)**: Its pentagonal and pyramidal structure is equivalent to the structure of IF$_7$ (Iodine Hepta Fluoride). IF$_7$ is one of the pioneer compounds which contains 2 electro negative elements.

![Structure of Iodine hepta fluoride(IF$_7$)](image2)

**Pentagonal prism**: Pentagonal Prism will exactly look like a part of structure that contains 2 pentagons and 5 squares.

![Pentagonal prism](image3)
Structure of Abstract water or pentagonal water\textsuperscript{[7]}: Pentagonal water or abstract water has structure close to structure of $S_5$.

![Structure of abstract water](image)

**STRUCTURAL INFORMATIONS**

- Every point of the structure contains a 5-cycle element of $S_5$ which is different from any other element of figure.
- Every point is connected with 4 points and these 4 points makes a square.
- Points of the pentagons are the rotations of one 5 cycle by an angle of $72.0^\circ$.
- Points on pentagons rotates with a fixed pattern which is given below:
Points on the square rotates with the pattern given below:

\[
\begin{align*}
& \text{(dabe)} \quad \text{(bcde)} \\
& \text{(cdabe)} \quad \text{(bcdae)}
\end{align*}
\]

**STRUCTURE OF A_5**

**Structure of Ashoka Chakra** [8]- Sketch of A_5 is similar to the Ashoka Chakra, which is the part of National Emblem and the part of Indian Flag. Ashoka chakra has 24 spokes which denotes 24 qualities of mankind. A_5 has 24 distinct 5- Cycles which are connected with center through three cycles.
FIGURE OF $A_5$
CONCLUSION

Figure of $S_5$ is similar to the deflated football or fullerene and the figure of $A_5$ similar to the Ashoka chakra which is the part of the National emblem

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BIBILOGRAPHY


WEB BIBILOGRAPHY


