

ACHARYA NARENDRA DEV COLLEGE



ELITE PROJECT – MATHEMATICAL SKETCH OF SYMMETRIC GROUP OF DEGREE 5 (S_5)

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ABSTARCT

In this project, we worked on the Symmetric Group S_5 , did its mathematical analysis and made the sketch of S_5 using its normal rotation. considering one of 5-cycle of S_5 we get 5 different cycles using its rotation .Again we have $5!/5 = 24$ choices of different cycles and hence 120 distinct 5- cycles , in this manner we plot these 120 elements and get a symmetric figure in 2 –dimensional plane whose points are interconnected in 3 –dimensional space. In the similar manner we plot the points of A_5 and get a structure is similar to Ashoka Chakra which is the part of National Emblem.

INTRODUCTION^[1]

Symmetric group is the set of all bijective functions from a finite set to itself, and forms a group under function composition.

Let $S = \{1,2,3,4,5\}$, then

$S_5 = \{f \mid f: S \rightarrow S \mid f \text{ is bijective}\}$, is a symmetric group under function composition.

CHARACTERISTICS^[2]

- Order of S_n is $n!$.
- $|S_5| = 5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120$.
- Maximum order of any element in S_5 is 6.
- S_5 is a group of even order and has 60 even permutations and 60 odd permutations.
- S_5 is non-abelian.
- S_5 has 156 subgroups.
- Set of all even permutations is a subgroup of S_5 called alternating group of degree 5.
- S_5 has 3 normal subgroups, which are $\{e\}$, A_5 and S_5 .
- S_5 is non-simple group as it has one non-trivial normal subgroup A_5 .
- $Z(S_5) = \{e\}$.

TABLE FOR ELEMENTS OF S_5 ^[1]

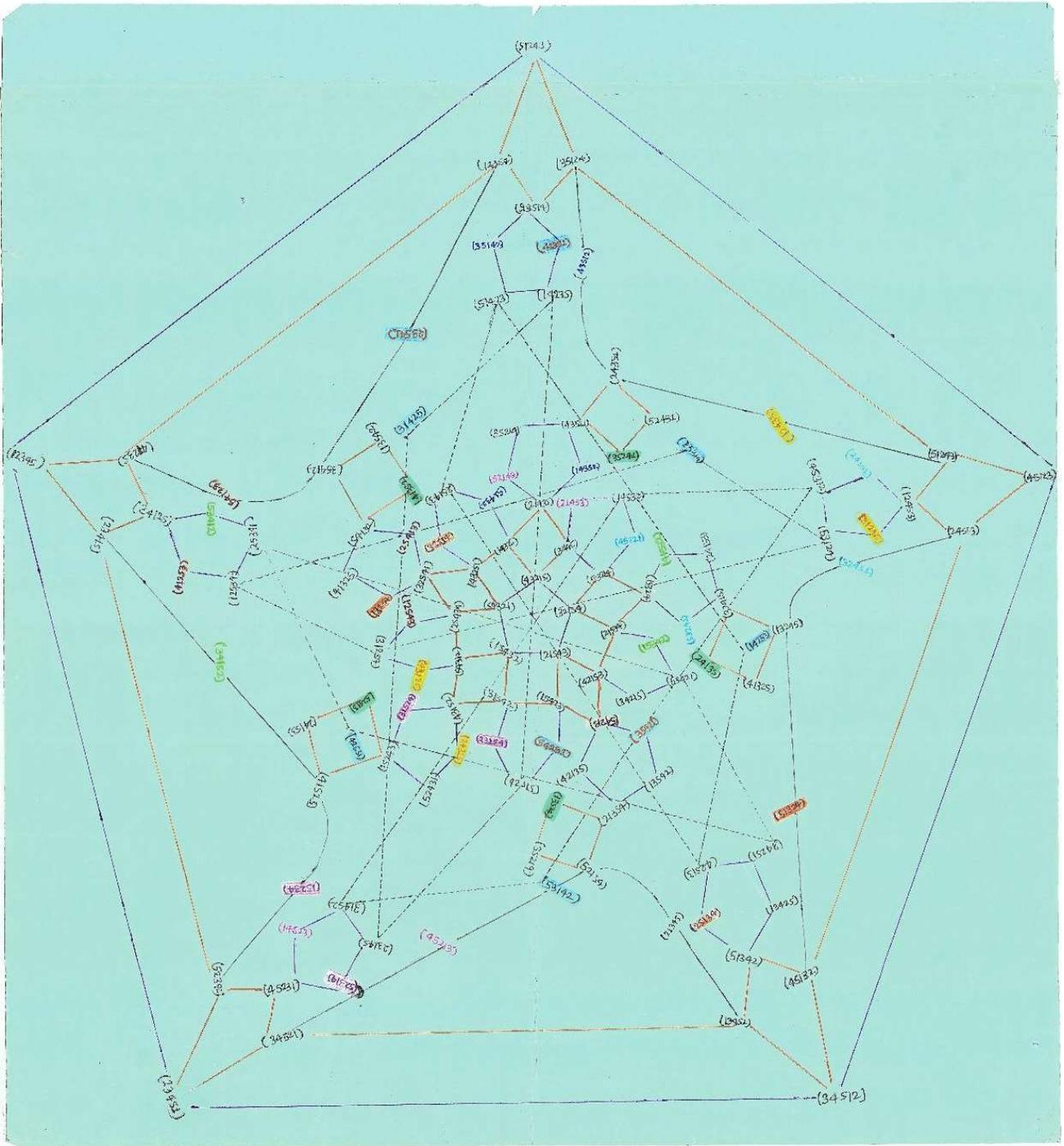
No. of the permutations (say p) based on the cyclic decomposition of a permutation in S_n then,

$$p = \frac{n!}{1^{K_1} 2^{K_2} \dots n^{K_n} K_1! K_2! \dots K_n!}$$

Where 1, 2, 3....., n is the length of disjoint cycles occurs in the decomposition and K_i 's denotes the occurrence of i-cycle.

Possible cyclic decomposition	Order of element	No. of such Permutation	Type of element
{1,1,1,1,1}	1	1	$I=\{e\}$
{1,1,1,2}	2	10	(a b)
{1,2,2}	2	15	(a b)(c d)
{1,1,3}	3	20	(a b c)
{2,3}	6	20	(a b)(c d e)
{1,4}	4	30	(a b c d)
{5}	5	24	(a b c d e)

FIGURE OF S₅

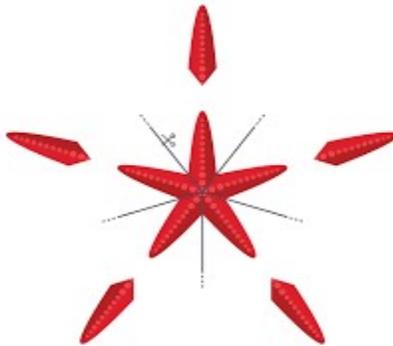


STRUCTURE OF S_5

- ❖ **Deflated football:** -According to the structure made by us we could easily observe that figure of S_5 looks like deflated football in 2-dimensional plane.

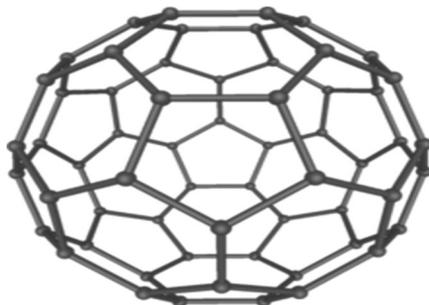


- ❖ Points at the corner are connected to the points on other side. This concludes that it is a 3-dimensional structure.
- ❖ As in the structure of football, pentagons are connected with hexagons but in our structure every pentagon is connected with a square or rhombus. This makes structure of S_5 different from football.
- ❖ **Starfish**^[3]: - Outer boundary figure looks like a Starfish. A Starfish has five legs. There is an angle of 72° between two consecutive legs which gives us a structure like symmetry of pentagons.



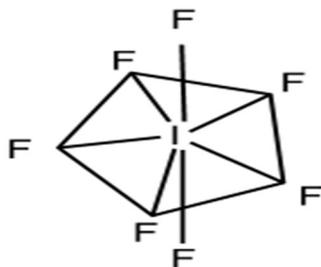
Symmetrical figure of Starfish

- ❖ **Structure of fullerene** ^[4]: - Chemists will definitely say that it is the structure of their well-known carbonic compound called “*fullerene*”. Fullerene is a well-known and very famous carbonic compound it is often called C₆₀.



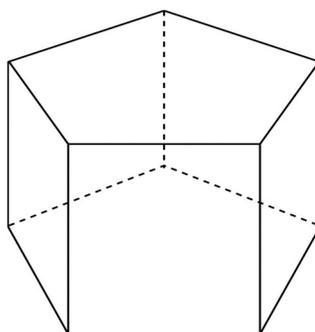
Structure of Fullerene

- ❖ **Structure of IF₇** (Iodine Hepta Fluoride) ^[5]: - Its pentagonal and pyramidal structure is equivalent to the structure of IF₇ (Iodine Hepta Fluoride). IF₇ is one of the pioneer compounds which contains 2 electro negative elements.



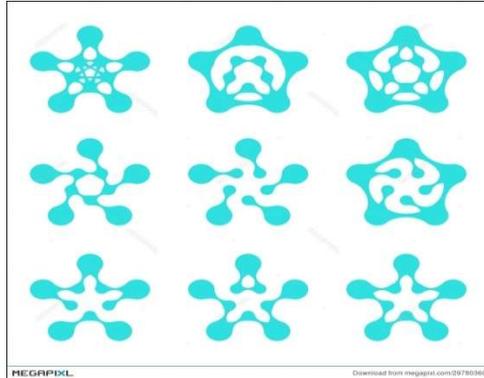
Structure of Iodine hepta fluoride(IF₇)

- ❖ **Pentagonal prism** ^[6]: - Pentagonal Prism will exactly look like a part of structure that contains 2 pentagons and 5 squares.



Pentagonal prism

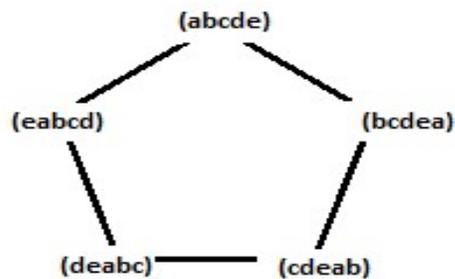
- ❖ **Structure of Abstract water or pentagonal water** ^[7]: -Pentagonal water or abstract water has structure close to structure of S_5 .



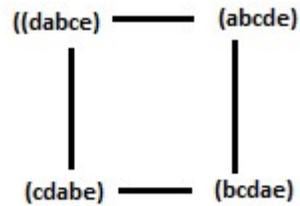
Structure of abstract water

STRUCTURAL INFORMATIONS

- ✓ Every point of the structure contains a 5-cycle element of S_5 which is different from any other element of figure.
- ✓ Every point is connected with 4 points and these 4 points makes a square.
- ✓ Points of the pentagons are the rotations of one 5 cycle by an angle of $72.^\circ$
- ✓ Points on pentagons rotates with a fixed pattern which is given below:



✓ Points on the square rotates with the pattern given below:



STRUCTURE OF A_5

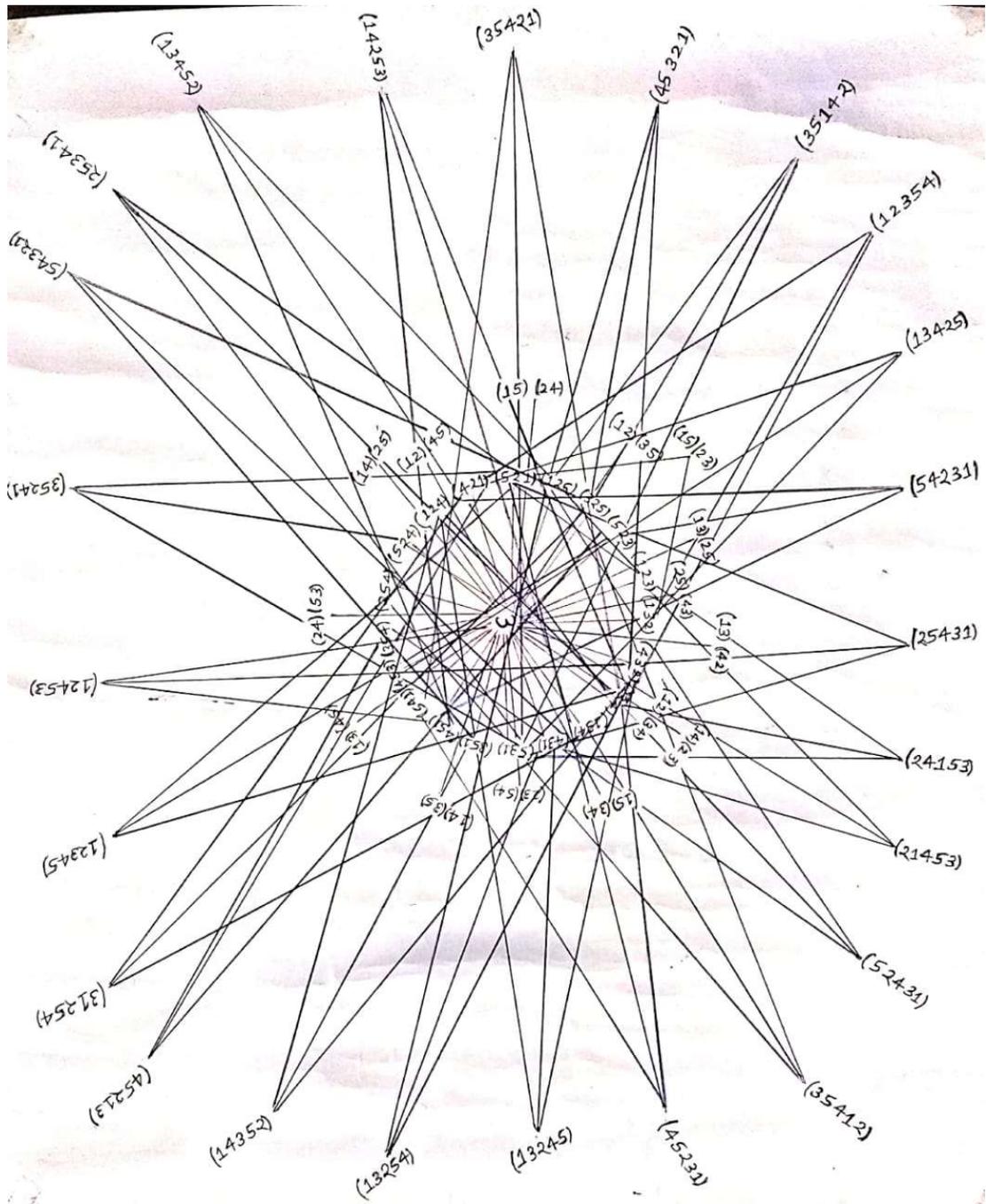
Structure of Ashoka Chakra ^[8] -Sketch of A_5 is similar to the Ashoka Chakra, which is the part of National Emblem and the part of Indian Flag. Ashoka chakra has 24 spokes which denotes 24 qualities of mankind.

A_5 has 24 distinct 5- Cycles which are connected with center through three cycles.



Ashoka Chakra

FIGURE OF A_5



CONCLUSION

Figure of S_5 is similar to the deflated football or fullerene and the figure of A_5 similar to the Ashoka chakra which is the part of the National emblem

ACKNOWLEDGEMENT

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- [8] https://en.wikipedia.org/wiki/Ashoka_Chakra