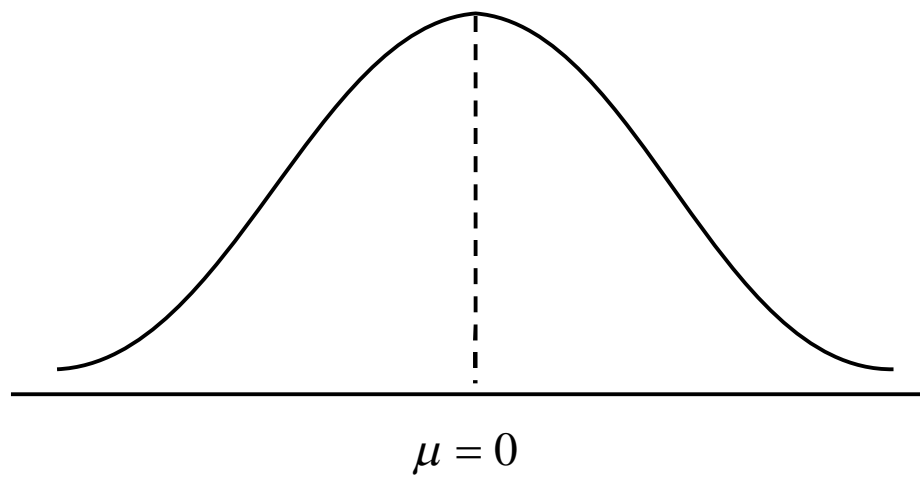


Normal Distribution

Introduction

The normal distribution is the most important continuous distribution in the entire field of statistics. Its graph called the normal curve is the bell-shaped curve



Definition The probability density function of the normal random variable with mean μ and variance σ^2 is given by

$$n(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \quad -\infty < x < \infty$$

Properties

- The graph of the probability function for any normal random variable is bell-shaped and symmetric about the vertical axis through the mean .
- The curve has its points of inflection at

$$x = \mu \pm \sigma$$

- The normal curve approaches the horizontal axis asymptotically as we proceed in either direction away from the mean.
- The total area under the curve and above the horizontal axis is equal to 1, since a normal curve is the graph of the probability function for a continuous random variable X .

- The probability that X assumes any specific value is 0, and $P(a < x < b)$ is calculated by finding the area under the graph of $f(x)$ between points a and b
- The mode, which is the point on the horizontal axis where the curve is a maximum, occurs at $x = \mu$.

Definition The normal distribution with $\mu = 0$ and $\sigma = 1$ is referred to as the standard normal distribution.

If in a given problem $\mu \neq 0$ and $\sigma \neq 1$ then there is the need to standardise the distribution by means of the transformation

$$Z = \frac{x - \mu}{\sigma}$$

Example 4.1

Find the probability that a random variable having the standard normal distribution will take on a value:

- a. less than 1.72
- b. less than -2.56
- c. between 0.87 and 1.28
- d. between -0.34 and 0.62

Example 4.2

Given a normal distribution with $\mu = 50$ and $\sigma = 10$, find the probability that x assumes a value between 45 and 62.

Example 4.3

A certain type of storage battery lasts on the average 3.0 years, with a standard deviation of 0.5 years. Assuming that the battery lives are normally distributed, find the probability that a given battery will last less than 2.3 years.

Normal approximation to the binomial distribution

The normal distribution can be used to approximate the binomial distribution when n , the sample size is large and p the probability of a success is close to zero.

Theorem If X is a binomial random variable with mean $\mu = np$ and variance $\sigma^2 = npq$, then the limiting form of the distribution of

$$z = \frac{x - np}{\sqrt{npq}} \quad n \rightarrow \infty$$

Example 4.4

If 20% of items in a large consignment are defectives, what is the probability that in a lot of 100 randomly chosen items for inspection,

- a. exactly 15 will be defective?
- b. at most 15 will be defective?