

# ECO 201: ELEMENTS OF MICROECONOMICS

## SAMPLE QUESTIONS AND ANSWERS

### Market Structures

1) Faced with two distinct demand functions  $Q_1 = 24 - 0.2P_1$  and  $Q_2 = 10 - 0.05P_2$  where  $TC = 35 + 40Q$ . What prices will the firm charge

- with discrimination and
- without discrimination

### **Solution**

a)  $Q_1 = 24 - 0.2P_1$  make P the subject  $P_1 = 120 - 5Q_1$

$$TR_1 = P_1Q_1 = (120 - 5Q_1)Q_1 = 120Q_1 - 5Q_1^2$$

$$MR_1 = \frac{\partial TR_1}{\partial Q_1} = 120 - 10Q_1$$

The firm will maximize profits where

$$MC = MR_1 = MR_2$$

$$\Rightarrow TC = 35 + 40Q$$

$$MC = \frac{\partial TC}{\partial Q} = 40$$

When  $MC = MR_1$

$$40 = 120 - 10Q_1$$

$$\Rightarrow Q_1 = 8$$

When  $Q_1 = 8$

$$P_1 = 120 - 5(8) = 80$$

In the second market, with

$$Q_2 = 10 - 0.05P_2$$

$$P_2 = 200 - 20Q_2$$

$$TR_2 = P_2Q_2 = (200 - 20Q_2)Q_2 = 200Q_2 - 20Q_2^2$$

$$MR_2 = \frac{\partial TR_2}{\partial Q_2} = 200 - 40Q_2$$

When  $MC = MR_2$

$$40 = 200 - 40Q_2$$

$$\Rightarrow Q_2 = 4$$

When  $Q_2 = 4$ ,  $P = 200 - 20(4) = 120$

b) If the producer does not discriminate,  $P_1 = P_2 = P$  and the two demand function can be combined.

$$Q = Q_1 + Q_2 = 24 - 0.2P + 10 - 0.05P$$

$$\Rightarrow Q = 34 - 0.25P$$

Thus  $P = 136 - 4Q$

$$TR = PQ = (136 - 4Q)Q = 136Q - 4Q^2$$

$$MR = 136 - 8Q$$

At the profit maximum level

$$MC = MR \quad \Rightarrow 40 = 136 - 8Q$$

$$Q = 12$$

At  $Q = 12$       $P = 136 - 4(12)$

$$P = 88$$

2) Given the demand curve of the monopolist,  $Q = 50 - 0.5P$  and the cost function to be

$$TC = 50 + 40Q$$

a) MR

$$P = 100 - 2Q$$

The goal of the monopolist is to maximize profit

$$\Pi = R - C \text{ i.e. } \Pi = TR - TC$$

$$TR = PQ = (100 - 2Q)Q = 100Q - 2Q^2$$

$$MR = \frac{\partial TR}{\partial Q} = 100 - 4Q$$

b) MC

$$TC = 50 + 40Q$$

$$MC = \frac{\partial TC}{\partial Q} = 40$$

c) We equate MR and MC

$$MR=MC$$

$$100 - 4Q = 40$$

$$Q = 15 \text{ max output}$$

d) Substituting  $Q=15$

$$P = 100 - 2(15) = 70$$

$$e) \Pi = TR - TC = 1050 - 650 = 400$$

Taking second order derivative

$$\text{from } \frac{\partial TR}{\partial Q} = 100 - 4Q \text{ we have } \frac{\partial^2 TR}{\partial Q^2} = -4$$

Clearly  $-4 < 0$ . This is maximum profit

3) Given the demand function of a monopolist as  $3P=90 - 2.25Q$  and his average cost (AC) function as  $3AC-90/Q=27+0.9Q$

- a. Find the output level,  $Q$ , which gives
  - i. Maximum revenue
  - ii. Minimum AC
  - iii. Maximum profit
- b. Find the value of the
  - i. Maximum revenue
  - ii. Minimum AC
  - iii. Maximum profit
- c. When a lump sum tax of 30 is imposed by the government. Find the new output and the profit
- d. Sketch the diagram for the type of profit.

### Solution

a)

$$i) \frac{3P}{3} = \frac{90}{3} - \frac{2.25Q}{3}$$

$$P = 30 - 0.75Q$$

$$TR = P \times Q$$

$$TR = (30 - 0.75Q)Q$$

$$TR = 30Q - 0.75Q^2$$

The first order derivative should be zero.

$$\frac{dTR}{dQ} = 0$$

$$MR = \frac{dTR}{dQ} = 30 - 1.5Q = 0$$

$$30 - 1.5Q = 0$$

$$\frac{30}{1.5} = \frac{1.5Q}{1.5}$$

$$Q = 20$$

$$\frac{d^2TR}{dQ^2} = -1.5 < 0 \text{ Hence relative maximum of TR curve}$$

$\therefore Q = 20$  gives maximum revenue

$$\text{ii) } 3AC = \frac{90}{Q} = 27 + 0.9Q$$

$$\frac{3AC}{3} = \frac{27}{3} + \frac{0.9Q}{3} + \frac{90}{3}$$

$$AC = 9 + 0.3Q + \frac{90}{3Q}$$

$$AC = 9 + 0.3Q + \frac{30}{Q}$$

$$AC = 9 + 0.3Q + 30Q^{-1}$$

$$\frac{dAC}{dQ} = 0$$

$$\frac{dAC}{dQ} = 0.3 - 30Q^{-2} = 0$$

$$0.3 - \frac{30}{Q^2} = 0$$

$$0.3 = \frac{30}{Q^2}$$

$$\frac{0.3Q^2}{0.3} = \frac{30}{0.3}$$

$$Q^2 = 100$$

$$Q = \sqrt{100}$$

$$Q = 10$$

$$\frac{d^2 AC}{dQ^2} = 60Q^{-3} > 0 \text{ or } \frac{60}{Q^3} > 0 \text{ (0.5) Hence relative minimum}$$

$\therefore Q = 10$  gives minimum AC

$$\text{iii) } \pi = TR - TC$$

$$TC = AC \times Q$$

$$TC = \left( 9 + 0.3Q + \frac{30}{Q} \right) Q$$

$$TC = 9Q + 0.3Q^2 + \frac{30Q}{Q}$$

$$TC = 9Q + 0.3Q^2 + 30 \text{ (0.5)}$$

$$\pi = TR - TC$$

$$\pi = (30Q - 0.75Q^2) - (9Q + 0.3Q^2 + 30) \text{ (0.5)}$$

$$\pi = 30Q - 0.75Q^2 - 9Q - 0.3Q^2 - 30$$

$$\pi = 21Q - 1.05Q^2 - 30 \text{ (0.5)}$$

$$\frac{d\pi}{dQ} = 21 - 2.1Q = 0 \text{ (0.5)}$$

$$\frac{21}{2.1} = \frac{2.1Q}{2.1}$$

$$\frac{d^2 \pi}{dQ^2} = -2.1 < 0 \text{ relative maximum}$$

$\therefore Q = 10$  gives maximum profit

**Alternatively**

$$MR = MC$$

$$MR = \frac{dTR}{dQ} = 30 - 1.5Q$$

$$MC = \frac{dTC}{dQ} = 9 + 0.6Q$$

$MR = MC$  Maximum profit level of output condition

$$30 - 1.5Q = 9 + 0.6Q$$

$$30 - 9 = 0.6Q + 1.5Q$$

$$\frac{21}{21} = \frac{2.1Q}{2.1}$$

$$Q = 10$$

$$\frac{dMR}{dQ} = -1.5 < 0 \text{ Relative maximum}$$

$$\frac{dMR}{dQ} = 0.6 > 0 \text{ Relative minimum}$$

$\therefore Q = 10$  gives maximum profit

**3) b.**

$$\text{i) } TR = 30Q - 0.75Q^2 = 30(20) - 0.75(20)^2$$

$$TR = 600 - 300 = 300 \text{ Maximum revenue}$$

$$\text{ii) } AC = 9 + 0.3Q + \frac{30}{Q} = 9 + 0.3(10) + \frac{30}{10}$$

$$AC = 9 + 3 + 3 = 15 \text{ Minimum AC}$$

$$\text{iii) } \pi = 21Q - 1.05(Q^2) - 30$$

$$\pi = 21(10) - 1.05(10)^2 - 30$$

$$\pi = 210 - 1.05(100) - 30$$

$$\pi = 210 - 105 - 30$$

$$\pi = 75 \text{ Maximum profit}$$

**3) c.**

$$TC_i = 9Q + 0.3Q^2 + 30 + T$$

But lump sum tax (T) = 30

$$TC_i = 9Q + 0.3Q^2 + 30 + 30$$

$$TC_i = 9Q + 0.3Q^2 + 60$$

$$\pi = TR - TC_t$$

$$\pi = (30Q - 0.75Q^2) - (9Q + 0.3Q^2 + 60)$$

$$\pi = 30Q - 0.75Q^2 - 9Q - 0.3Q^2 - 60$$

$$\pi = 21Q - 1.05Q^2 - 60$$

$$\frac{d\pi}{dQ} = 21 - 2.1Q = 0$$

$$21 - 2.1Q = 0$$

$$\frac{21}{2.1} = \frac{2.1Q}{2.1}$$

$$Q = 10$$

$$\frac{d^2\pi}{dQ^2} = -2.1 < 0 \text{ relative maximum}$$

But  $\pi = 21Q - 1.05Q^2 - 60$

$$\pi = 21(10) - 1.05(10)^2 - 60$$

$$\pi = 210 - 105 - 60$$

$$\pi = 45$$

3) d.

The monopolist price is found by substituting  $Q=10$  into the demand- price function

$$P = 30 - 0.75(10)$$

$$P = 30 - 7.5 = 22.5$$

$$P = 22.5$$

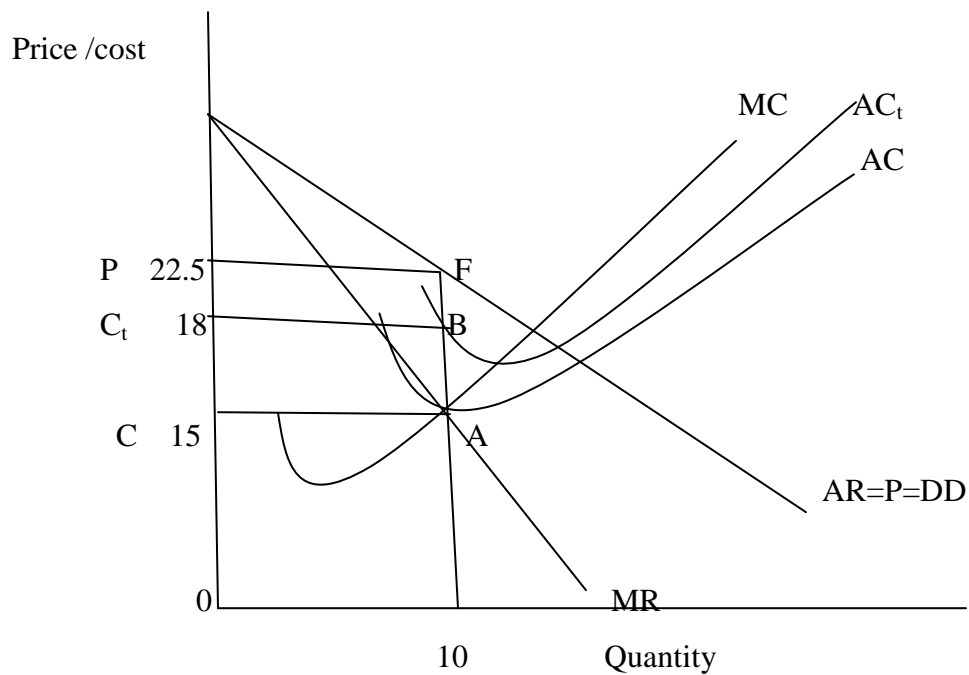
$$AC_t = \frac{TC_t}{Q} = \frac{9Q}{Q} + \frac{0.3Q^2}{Q} + \frac{60}{Q}$$

$$AC_t = 9 + 0.3Q + \frac{60}{Q}$$

$$AC_t = 9 + 0.3(10) + \frac{60}{10} = 9 + 3 + 6 = 18$$

$AC_t = 18$  value of  $AC_t$  minimum after lump sum Tax

That is,  $AC = 15$  before tax and  $AC_t = 18$  after tax



Area IPCAF represents excess profit before the imposition of the lump sum tax

Area  $PC_tBF$  represents excess profit after imposition of tax

**Conclusion:** the imposition of the lump sum tax led to a reduction of the excess profits of the monopolist from 75 to 45, but left the equilibrium position of the monopolist unchanged since it is the Fixed Cost (FC) that is affected and not the MC.

### Disclaimer:

These are just sample questions to help you in your studies and do not necessarily represent the entire syllabus nor the difficult level of questions in Elements of Microeconomics.