Negative Binomial Distribution

Introduction

Let us consider an experiment in which the properties are the same as those listed for a binomial experiment, with the exception that the trials will be repeated until a fixed number of successes occur.

- Therefore, instead of finding the probability of *x* successes in *n* trials, where *n* is fixed, we now are interested in the probability that the *kth* success occurs on the *xth* trial

- Experiments of this nature are called Negative binomial experiments

- For instance, we may be interested in the probability that the tenth driver observed on a roadblock will be the third to wear a seatbelt.

- The probability that the fifth person to hear a rumour will be the first one to believe it,

- or the probability that a burglar will be caught for the second time on his or her ninth job.

- **Definition** The number of trials *X* to produce k successes in a negative binomial experiment is called a negative binomial variable.

The negative binomial random distribution is based on an experiment satisfying the following conditions:

- The experiment consists of a sequence of independent trials.
- Each trial can result in either a success or a failure.

- The probability of success is constant from trial to trial.
- The experiment continuous until a total of *r* successes have been observed, where *r* a is specified positive integer.

- The random variable of interest is *X*, the number of failures that precede the *rth* success.

- *X* is called a negative binomial random variable because in contrast to the binomial random variable, the number of success is fixed and the number of trials is random.

- **Definition** If repeated independent trials can result in a success with probability *p* and a failure with probability $1-p$, then the probability distribution of the random variable *X*, the number of the trial on which the *kth* success occurs is given by

where $$b^*(x;k,p) = {}^{x-1}C_{k-1}\,p^k(1-p)^{x-k}$$

$$x = k, k+1, k+2, \cdots$$

In statistics, negative binomial distributions are also referred to as binomial waiting-time distributions or Pascal distributions.

Example 3.8

If the probability is 0.40 that a driver is observed to wear a seatbelt, what is the probability that the fifteenth driver observed will be the third to wear a seatbelt?

Example 3.9

Find the probability that a person tossing three coins will get either all heads or all tails for the second time on the fifth toss.