

# COURSE MANUAL-MAT 363 ODE

## Course Information

Title: Ordinary Differential Equations

Code: MAT 363

Credits: 3

Entry Requirements: **Students must take MAT 111 Algebra I and MAT 235 Calculus I**

## Instructor(s) Information

Name: Oray

Position: Lecturer

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## Introduction:

The study of differential equations is a beautiful application of the ideas and techniques of calculus to our everyday lives. Indeed, it could be said that calculus was developed mainly so that the fundamental principles that govern many phenomena could be compressed in the language of differential equations. The purpose of this course is to provide an introduction to ordinary differential equations and their applications for students of mathematics and mathematics education in tertiary institutions. It is designed to provide for students a broad perspective on the subject, to illustrate direct variety of phenomena encompassed by it and to impart a working knowledge of the most important techniques of analysis of the solution of the equation. Knowledge of elementary algebra and introductory calculus in the universities and colleges is therefore assumed.

## Subject content

Fundamental concepts of differential equations; Separable differential equations; Homogenous differential equations of the first order; Exact differential equations; Linear differential equations of the first order; Applications of first order differential equations. Second order differential linear equations; Linear independence and the Wronskian; Homogeneous Linear equations; Non-Homogenous linear equations. Power series solutions. Boundary and Eigen-value problems.

## Connection with other courses:

This course has a lot of connections with the course you took in Calculus, Linear Algebra and Introductory Analysis. Most mathematical models and applications of mathematics in industry and technology are based on differential equations.

**Learning objectives:****By the end of the course, students should be able to:**

- (i) identify different types of first order differential equations and use appropriate methods to solve them
- (ii) solve second order linear differential equations
- (iii) explain linear independence with reference to the Wronskian
- (iv) differentiate between homogeneous and non-homogeneous differential equations
- (v) determine general and particular solutions of homogeneous and non-homogeneous differential equations
- (vi) find power series solutions
- (vii) solve boundary and eigen-value problems

**Literature and materials**

## Compulsory study texts:

- (i) Asiedu-Addo, S., Awanta, K. & Ampiah; E. (2006); Elements of Ordinary Differential Equations, City Printers, Accra.

## Supplementary study texts:

- (ii) Steward J, Lothar Redlin, Saleen Watson (2002) Pre-Calculus Mathematics for Calculus Brooks/Cole
- (iii) Robert Adams (2005) Calculus a complete course Addison Wesley
- (iv) Any other relevant calculus text

## Materials used:

- (i) Scientific calculators
- (ii) Computers
- (iii) Note books

**Course Schedule**

<b>Week</b>	<b>Content Topics</b>	<b>Learning activities</b>	<b>Pre-Lesson Preparation</b>
1	Differential equations and their classifications.  First order differential equations and initial value problems (IVP).	In groups, students discuss and explain what a differential equation is.  Students classify differential equations into:- ODE & PDE; linear & non-linear; order of equations. Students	Read pages 3-4 and 10-12 of the main textbook and work through Examples 2.1, 2.2 and 2.3 of chapter 2 of course textbook.

		<p>discuss solutions to differential equations in their small groups.</p> <p>Hands-on activity on Graphic Calculus to demonstrate direction fields.</p> <p>Students to deduce the first order DE from the nth order equation and discuss solutions of equations in small groups.</p> <p>Relate initial conditions with constraints and restraints as in linear programming and define IVP</p>	
2	Separable Differential Equations	<p>In small groups, students deduce separable DE's and determine their solutions and confirm by applying the software Derive 6.</p> <p>Use Graphic Calculus to demonstrate the concept of orthogonal trajectories. Students to work problems on orthogonal trajectories.</p>	<p>Study sections 2.4 and 2.5 of the main course textbook and work through Examples 2.5, 2.6, 2.7, 2.8 and 2.9.</p> <p><b>Submission of Tutorial Set 1</b></p>
3	Separable Differential Equations-cont'd	<p>Apply separation of variables method to solve problems in Biology, Economics, Chemistry and Medicine. Students work through problems in chapter 6 in groups.</p>	<p>Study chapter 6 of the course textbook and work through examples 6.1 to 6.4. Pay special attention to section 1.3.</p>
4	Homogeneous differential equations and their solutions	<p>Students discuss with the instructor the concept of homogeneous function after which they work through examples 3.1</p>	<p>Students to read course textbook chapter 3, page 32 and pay special attention to the different types of homogeneous equations and</p>

		<p>through 3.5 in small groups.</p> <p>Students take active participation in the proof of the theorems of the solution of homogeneous equations and apply them in solving problems.</p> <p>Students to differentiate between types of homogeneous equations and find appropriate solutions to them.</p>	<p>their method of solving them.</p> <p><b>Submission of Tutorial Set 2</b></p>
5	Exact differential equations and integrating factors	<p>Starting from the definition of total differential, students deduce exact DE in small groups.</p> <p>Students to proof the conditions for exactness of equations and show that given DE's are exact.</p> <p>Students solve problems on exact equations in groups and then each group presents its solution to the whole class.</p> <p>Students to distinguish between various types of integrating factors and solve them accordingly.</p> <p>Hands-on activity on Derive 6 to show the graphs of the solutions obtained.</p>	<p>Review total differential of functions of several variables. Study chapter 4 of the course textbook and work through examples 4.5, 4.6 and 4.7.</p> <p><b>Submission of Tutorial Set 3</b></p>
<p><b>One hour quiz will be taken by all students taken this course after weeks 2-4 topics have been treated</b></p>			

6	<p>First order linear differential equations and equations reducible to them</p>	<p>Students to deduce a first order linear DE from the general definition given in chapter 1.</p> <p>Students to identify first order linear equations from a number of equations.</p> <p>Students to take active participation in the proof of the theorems of the solution of first order linear equations and apply them in solving problems.</p> <p>In groups, students determine conditions under which Bernoulli's equations could become linear equations and solve Bernoulli's equations.</p>	<p>Study the properties of functions that can serve as solutions to first order linear equations. Read section 5.3 of the course textbook and work through examples 5.1 and 5.2.</p> <p><b>Submission of Tutorial Set 4</b></p>
7	<p>Higher Order Linear Equations</p>	<p>From the first order linear DE and nth degree polynomial equations, students in groups will deduce the nth order linear DE.</p> <p>Students explain the difference between homogeneous and non-homogeneous DE's.</p> <p>From their linear algebra course, students will define linear operators and use it to write nth order linear DE in operator notion form.</p> <p>Students will discuss among themselves the theorem of existence and uniqueness of solutions of DE's and compare it to a</p>	<p>Study sections 8.1 and 8.2 of the main course textbook and study examples 8.3 to 8.6.</p> <p><b>Submission of Tutorial Set 5</b></p>

		<p>similar theorem in chapter 1.</p> <p>Students in small groups discuss linear dependence as learnt in linear algebra and apply it to linear dependence in DE's.</p> <p>Students to show on the blackboard whether two functions are linearly dependent or not.</p>	
8	Higher Order Linear Equations cont'd	<p>Students take active participation in the definition and explanation of the Wronskian of two or three functions.</p> <p>In small groups, students find the relation between the Wronskian and linear dependence.</p> <p>In groups, students will show that two or three functions form a fundamental set of solutions.</p> <p>Students will solve problems higher order DE's by the reduction of order method first individually and then in small groups.</p>	<p>Read section 8.3 of the main course textbook and note the relationship between linearly independent solutions and fundamental set of solutions. Work through examples 8.12 and 8.14 in the main course textbook.</p> <p><b>Submission of Tutorial Set 6</b></p>
<p><b>One hour quiz will be taken by all students taken this course after weeks 5-7 topics have been treated</b></p>			
9	<p>Homogeneous Linear Equations with constant coefficients</p> <p>Cauchy-Euler Equations</p>	<p>Students will define homogeneous DE's of the nth order and deduce their characteristic equations.</p> <p>Students will solve the</p>	<p>Read section 7.2 of the main textbook and take note of the properties of the three cases of the roots of the characteristic equations.</p> <p>Work through examples 7.1</p>

		<p>three types of higher order linear DE's. This will be done in groups of three. Each group will then give a presentation.</p> <p>Students will systematically list the steps through which the solution of an nth order linear equation could be found.</p> <p>Students to find ways of identifying Cauchy-Euler equations. In groups, students will discuss the transformation that will change a Cauchy-Euler equation to an equation with constant coefficient and use it to solve some given problems.</p>	<p>to 7.3.</p> <p><b>Submission of Tutorial Set 8</b></p>
10	<p>Non-Homogeneous Linear Equations</p>	<p>Students to discover that to every non-homogeneous equation, there is an associated homogeneous equation and deduce that the general solution of a non-homogeneous equation is given by <math>y = y_h + y_p</math>, where <math>y_h</math> is the homogeneous solution and <math>y_p</math> is the particular solution to the equation.</p> <p>Students in an active mood, pay attention to the derivation of the method of undetermined coefficients in solving non-homogeneous linear equations.</p> <p>There will be hands-on</p>	<p>Read section 7.3 of the main textbook and work through examples 7.4 and 7.5.</p>

		activity on Derive 6 to solve non-homogeneous equations using the method of undetermined coefficients.	
11	Non-Homogeneous Linear Equations-cont'd	<p>Students to establish that the method of variation of parameters is the most general method for solving a non-homogeneous equation.</p> <p>In groups, students discuss from first principles the proof of the method of variation of parameters and solve problems on them.</p>	Work through theorem 10.2 on page 162 and study examples 10.3 and 10.4
12	Power Series Solutions about an Ordinary Point	<p>Students to discuss the need for power series method to find solutions to DE's and to explain the terms, convergence and divergence of series.</p> <p>Students will list some test used to determine the convergence of power series and describe Taylor series.</p> <p>Students will distinguish between ordinary points, singular points, regular points and irregular points and determine the power series solutions about an ordinary point.</p>	<p>Read sections 11.1, 11.2 and 11.3 and distinguish between ordinary points and singular points. Classify singular points into regular and irregular singular points and work through examples 11.1 and 11.4.</p> <p><b>Submission of Tutorial Set 9</b></p>
13	Method of Frobenius	Students will consider the three cases of how to find the power series solution about a regular singular point by the method Frobenius.	<p>Read section 11.4 of the main course textbook and study the given examples.</p> <p><b>Submission of Tutorial Set 10</b></p>

		In small groups students will solve problems involving regular singular points.	
<b>One hour quiz will be taken by all students taken this course after weeks 8-12 topics have been treated</b>			
14	Boundary Value Problems (BVP)  Eigen-Value Problems (EVP)	Students to distinguish between IVP's and BVP's and show that BVP's may have a unique solution, infinitely many solutions or no solutions. Students to solve problems involving BVP's.  Students in small groups discuss eigen-values and eigen functions and explain that eigen-functions are non-trivial solutions of DE's.  Students to establish that both BVP's and EVP's apply only to equations of at least order two.	Read chapter 12 of the main course textbook and study the given examples.

### ASSIGNMENTS:

All assignments are taken from the course book: Asiedu-Addo, S., Awanta, K. & Ampiah; E. (2006); Elements of Ordinary Differential Equations, (Revised Ed), City Printers, Accra. **Assignments and tutorial presentations are due as given above. Late work is unacceptable.**

### MODE OF ASSESSMENT

Attendance and class contribution	5%
Assignments (group or individual)	10%
Quizzes	25%
Final Exam	60%

### Grading Policies:

**A** = 80-100, **B+** = 75-79, **B** = 70-74, **C+** = 65-69, **C** = 60-64, **D+** = 55-59, **D** = 50-54, Below 50 = **E**

**COURSE POLICIES:**

Any violation of the university rules relating to courses will result in a failing grade on the assignment and possible university disciplinary action.

**CODE OF CONDUCT:**

- Attendance and active participation are **required** in this class. You are expected to read all the assigned material before class and attend every class session fully prepared to participate in discussions and activities. Lateness to lectures will not be entertained.
- Do assigned homework on time, show eagerness, aggressiveness and enthusiasm towards the course
- Switch off cell phones before entering the lecture theater
- Be courteous and considerate to your other classmates and to the lecturer. Harassment in any form will not be tolerated.
- Students with disabilities will be appropriately accommodated. Please let me know at the beginning of the semester if you need particular accommodations.
- Mathematics is a cumulative subject. Do not expect to understand subsequent material if you have not mastered current material

**DRESS CODE:**

- Men           Decent Trousers (preferably dark), white shirt and tie.
- Women       Decent Trousers or skirt (preferably dark), white shirt and tie or morfla.

**PLAGIARISM:**

Plagiarized work will receive a score of zero on the assignments. Plagiarism may also warrant a failing grade in the course and /or university disciplinary action.