Sampling Distribution of the Mean

The Z Distribution

Introduction

- If a random sample of size *n* is taken from a population having the mean μ and the variance σ^2 then $\overline{x}$ is a value of a random variable whose distribution has the mean μ.
- Thus $\mu_x = \mu$ or $\overline{x} = \mu$

- For samples from infinite populations the variance of this distribution is given by

$$\sigma_x^2 = \frac{\sigma}{n}$$

- The reliability of the mean as an estimate of μ is often measured by $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

also called the standard error of the mean.

Theorem: The Central Limit Theorem (CLT)

If $\overline{x}$ is the mean of a random sample of size *n* taken from a population having the mean μ and the finite variance σ^2, then

$$Z = \frac{\overline{x} - \mu}{\sigma / \sqrt{n}}$$

is the value of a random variable whose distribution function approaches that of the standard normal distribution as $n \to \infty$.

Example 5.1

When a batch of a certain chemical product is prepared, the amount of a particular impurity in the batch is a random variable with mean 4.0*g* and standard deviation 1.5*g*. If 50 batches are independently prepared what is the probability that the sample average impurity is between 3.5*g* and 3.8*g*?