3.

Problem 11.44P (HRW)

A pulsar is a rapidly rotating neutron star that emits radio pulses with precise synchronisation, one such pulse for each rotation of the star. The period $T$ of rotation is found by measuring the time between pulses. At present, the pulsar in the central region of the Crab nebula has a period of rotation of $T=0.033$ s, and this period is observed to be increasing at the rate of $1.26 \times 10^{-5}$ s/y.

(a) What is the value of the angular acceleration in rad/s$^2$?

(b) If its angular acceleration is constant, how many years from now will the pulsar stop rotating?

(c) The pulsar originated in a supernova explosion seen in the year A.D. 1054. What was $T$ for the pulsar when it was born? (Assume constant angular acceleration since then.)

Solution:

(a)
Angular speed $\omega$ and period $T$ are related as 

$$\omega = \frac{2\pi}{T}.$$ 

Therefore,

$$\frac{d\omega}{dt} = -\frac{2\pi}{T^2} \frac{dT}{dt}.$$ 

We have been given that for the pulsar

$$\frac{dT}{dt} = 1.26 \times 10^{-5} \text{ s/y} = \frac{1.26 \times 10^{-5}}{3.156 \times 10^7} = 3.99 \times 10^{-13}. $$

Therefore, the angular acceleration of the pulsar is

$$\alpha = \frac{d\omega}{dt} = -\frac{2\pi}{0.033^2} \times 3.99 \times 10^{-13} \text{ rad/s}^2 = -2.30 \times 10^{-9} \text{ rad/s}^2.$$ 

(b)

The present angular speed of the pulsar is

$$\omega = \frac{2\pi}{0.033} \text{ rad s}^{-1} = 1.90 \times 10^2 \text{ rad s}^{-1}. $$

If the pulsar will continue to decelerate at this rate, it will stop rotating at time $t_f$. 
\[ t_f = \frac{\omega}{\alpha} = \frac{1.90 \times 10^2}{2.30 \times 10^{-9}} \text{ s} = 8.27 \times 10^{10} \text{ s}, \]

or

\[ t_f = \frac{8.27 \times 10^{10}}{3.156 \times 10^7} \text{ y} = 2621 \text{ y}. \]

(c)

As the pulsar was formed in 1054 A.D. its life till the present has been 949 y. Its period of rotation at the time of its birth

\[ T_i = (0.033 - 1.26 \times 10^{-5} \times 949) \text{ s} = 0.021 \text{ s}. \]