Sampling Distribution of the Mean
(The t- Distribution)

Introduction

Application of the theory of the preceding section requires knowledge of the population standard deviation σ. If *n* is large $(n \geq 30)$, this does not pose any problems even when is not known, as it is reasonable that in this case we substitute for it the sample standard deviation *s*.

Theorem: If $\bar{x}$ is the mean of a random sample of size *n* taken from a normal population having the same mean μ and the variance σ^2 then

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

is the value of a random variable having the t-distribution with the parameter

$$\nu = n - 1.$$

Properties of t Distribution

Let t_ν denote the density function curve for ν degrees of freedom.

1. Each t_ν curve is bell-shaped and centered at 0.
2. As ν increases, the spread of the corresponding t_ν decreases.
3. Each t_ν curve is more spread out than the standard normal curve.
4. As $\nu \to \infty$, the sequence of t_ν curves approaches the

Example 5.2

A random sample of size 25 from a normal population has the mean 47.5 and standard deviation 8.4. Does this information tend to support or refute the claim that the mean of the population is 42.5?