The Binomial Distribution

Introduction

- Many statistical problems deal with situations referred to as repeated trials.
- Repeated trials play a very important role in probability and statistics, especially when the number of trials is fixed, the probability of a success is the same for each trial, and the trials are all independent

- For example, we would like to know the probability that 9 out of 10 VCR's will run at least 1000 hours.
- The probability that 45 of 120 drivers stopped at a roadblock will be wearing seatbelts.
- The probability that 7 of 10 persons will recover from malaria
- The probability that 35 out of 47 candidates will pass the impending end of semester examination

- In each of these examples, we are interested in the probability of getting *x* successes in *n* trials.

- A binomial experiment is one that possesses the following properties:

- The experiment consists of *n* repeated trials
- There are only two possible outcomes for each trial. “Success” and “failure”.
- The probability of a success denoted by *p* remains constant from trial to trial.
- The repeated *n* trials are all independent.

Definition: A random variable *X* has a binomial distribution and it is referred to as a binomial random variable if its probability distribution is given by:

$$b(x;n,p) = {}^{n}C_{x}\, p^{x}(1-p)^{n-x}$$

$$x = 0, 1, 2, \cdots, n$$

for

Example 3.1

Each of five questions on a multiple-choice examination has four choices, only one of which is correct. What is the probability that:

a. a student will get exactly three answers correct?

b. a student will get at most three answers correct?

c. a student will get at least four correct

Example 3.2

Find the probability that on the street of Accra, 7 out of 10 tro-tro drivers will wear seatbelts at a roadblock if we can assume independence and the probability is 0.2 that any one of them will wear a seatbelt.

If a random variable takes on the values $x_1, x_2, \cdots x_k$ with probabilities $f(x_1), f(x_2), \cdots, f(x_k)$, ,its mathematical expectation is given by .

$$x_1 f(x_1) + x_2 f(x_2) + \cdots x_k f(x_k)$$

That is , $$\mu = \sum_{all.x} x f(x)$$

Theorem: For the random variable *X* that has a binomial distribution with parameter *p* and *n*, the mathematical expectation is given by

$$E[X] = \mu = np$$

Definition: The variance and the standard deviation of the binomial distribution with parameters *n* and *p* are respectively

$$\sigma^2 = np(1-p) \quad \text{and} \quad \sigma = \sqrt{np(1-p)}$$

Example 3.3

If 75% of all consultations handled by lecturer consultants at a computing centre involve programs with syntax errors, and *X* is the number of programs with syntax errors in 10 randomly chosen consultations, then the mathematical expectation is given by

$$E[X] = np = 10(0.75) = 7.5$$