Separable Differential Equations

Separable Differential Equations?

Consider the first order equation $\frac{dy}{dx} = f(x, y)$

We write this equation in the form

$$M(x, y)dx + N(x, y)dy = 0$$

In the event that m is a function of x only and N is a function of y only

$$M(x)dx + N(y)dy = 0$$

Definition A first order differential equation is said to have its variables separable if it can be written in the form

$$\frac{dy}{g(y)} = f(x)dx$$

Example: Determine if the following equation is separable $xyy' = 1$

The general solution of first order equation is determined by integrating both sides of that equation.

$$\int \frac{dy}{g(y)} = \int f(x)dx + c$$

Example 2.5

Find the general solution of the equation

$$\frac{dy}{dx} = xy^2$$

Example 2.6

Determine the general solution of the differential equation $xdy - ydx = 0$

Example 2.7

Solve the initial value problem

$$\begin{cases} 2x(y+1)dx - ydy = 0 \\ y(0) = -2 \end{cases}$$

Orthogonal Trajectories

Definition: Orthogonal trajectory to a curve is another curve that intersects it at right angles.

Suppose we have the one-parameter family of curves defined by

$$f(x, y, c) = 0$$

In certain applications it is important to be able to obtain a second family of curves given by

$$g(x, y, c) = 0$$

with the property that all intersections of the two families are orthogonal.

The two families are then said to be orthogonal trajectories of each other. This means that their slopes at the points of intersection are negative reciprocals.

Suppose the function $f(x, y, c) = 0$ is a solution of a First order differential equation.

Then the equation $$\frac{dy}{dx} = -\frac{1}{f(x, y)}$$

Has the solution $$g(x, y, c) = 0$$

with the property that all intersections of the Two families are orthogonal.

The procedure is therefore to find a differential equation $\frac{dy}{dx} = f(x, y)$ for

which family $g(x, y, c) = 0$ is a general solution and then obtain the orthogonal trajectories as solutions.

Example 2.8

Find the orthogonal trajectories of the family of curves $y = cx^2$.

Example 2.9

Find the orthogonal trajectories of the family of circles $x^2 + y^2 = c^2$.