Roll No. ....

B.Sc.- Physics(Hons) / III

(Mathematical Physics - III)

Time : 3 Hours | Maximum Marks : 50

Question No. 1 is **compulsory** & attempt **four** other questions, choosing at least one from each section.

Q 1 Answer any *four* : $4 \times 2\frac{1}{2} = 10$

(a) Solve

$$2x + 3y - 2z = 5$$
$$x - 2y + 3z = 5$$
$$4x - y + 4z = 1$$

(b) Show that if the Fourier transform of a real function $f(t)$ is real, then $f(t)$ is an even function of t, and if the Fourier transform of a real function $f(t)$ is purely imaginary, then $f(t)$ is an odd funtion of t.

(c) Find

$$L^{-1}\{ln(1 + \frac{1}{s})\}$$

(d) Write u as a linear combination of the polynomials $v = 2t^2 + 3t - 4$ and $w = t^2 - 2t - 3$ where $u = 4t^2 - 6t - 1$.

(e) Find the Fourier transform of the shifted impulse function $\delta(t - t_0)$

Section A

Q 2 (a) Show that the polynomials $(1-t)^3$, $(1-t)^2$, $(1-t)$ and 1 generate the space of polynomials of degree$\leq$ 3. 4

(b) Let U and W be the subspaces of $\mathbf{R}^4$ generated by 3, 3
$\{(1, 1, 0, -1), (1, 2, 3, 0), (2, 3, 3, -1)\}$ and $\{(1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)\}$ respectively. Find (i) $\dim(U + W)$, (ii) $\dim(U \cap W)$.

Q 3 (a) Let W be the space generated by the polynomials
$u = t^3 + 2t^2 - 2t + 1$, $v = t^3 + 3t^2 - t + 4$ and $w = 2t^3 + t^2 - 7t - 7$
Find a basis and the dimension of W. 6

(b) Find the coordinate vector of $v = (4, \ -3, \ 2)$ relative to the basis $\{(1,1,1), \ (1,1,0), \ (1,0,0)\}$. 4

Section B

Q 4 Find the rank of matrix A where: 10

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$$

Q 5 (a) Let u, v and w be independent vectors. Show that $u+w, u-w$ and $u-2v+w$ are also independent. 4

(b) Determine whether or not the vector $v = (2, -5, 4)$ belong to the subspace of R^3 spanned by $\alpha_1 = (2, -1, 1)$ and $\alpha_2 = (1, -3, 2)$. 6

Section C

Q 6 (a) Find the Fourier transform of the function $f(t) = e^{-|t|}$. 6

(b) Find 4

$$L\{\frac{1-e^{-t}}{t}\}$$

Q 7 (a) A particle is executing simple harmonic oscillations expressed by the differential equation

$$\frac{d^2x(t)}{dt^2} + \omega^2 x(t) = 0$$

with the initial condtions at $t = 0$:

$$x(0) = 0 \qquad \frac{dx(t=0)}{dt} = 0$$

where ω is a constant. Using the method of Laplace transforms find the equation of motion of the particle. 6

(b) Use convolution to find 4

$$f(t) = F^{-1}[\frac{1}{(1+j\omega)^2}]$$