Roll No. ....

B.Sc.- Physics(Hons) / III

(Mathematical Physics - III)

Time : 3 Hours Maximum Marks : 50

Question No. 1 is **compulsory** & attempt **four** other questions, choosing at least one from each section.

Q 1 Answer any *four* : $4 \times 2\frac{1}{2} = 10$

(a) Solve

$$2x + 3y - 2z = 5$$

$$x - 2y + 3z = 5$$

$$4x - y + 4z = 1$$

(b) Consider the following transformation of the Cartesian plane R^2:

$$T[x, y] = [x\cos\theta - y\sin\theta,\ x\sin\theta + y\cos\theta]$$

Is this a linear transformation ?

(c) Prove that the determinant of a Hermitian matrix is real.

(d) Write u as a linear combination of the polynomials $v = 2t^2 + 3t - 4$ and $w = t^2 - 2t - 3$ where $u = 4t^2 - 6t - 1$.

(e) Prove that every linear transformation maps the zero vector to the zero vector.

Section A

Q 2 (a) If T and S are nonsingular transformations, then prove that 4

$$(TS)^{-1} = S^{-1}T^{-1}$$

(b) Let U and W be the subspaces of $\mathbf{R}^4$ generated by 3 + 3
$\{(1, 1, 0, -1), (1, 2, 3, 0), (2, 3, 3, -1)\}$ and $\{(1, 2, 2, -2), (2, 3, 2, -3), (1, 3, 4, -3)\}$ respectively. Find (i) $\dim(U + W)$, (ii) $\dim(U \cap W)$.

Q 3 (a) Let W be the space generated by the polynomials
$u = t^3 + 2t^2 - 2t + 1,\ \ v = t^3 + 3t^2 - t + 4$ and $w = 2t^3 + t^2 - 7t - 7$
Find a basis and the dimension of W. 6

(b) Find the coordinate vector of $v = (4,\ \ -3,\ 2)$
relative to the basis $\{(1,1,1),\ (1,1,0),\ (1,0,0)\}$. 4

Section B

Q 4 Find the rank of matrix A where: 10

$$A = \begin{pmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{pmatrix}$$

Q 5 (a) Let u, v and w be independent vectors. Show that $u+w, u-w$ and $u-2v+w$ are also independent. 4

(b) In the space of polynomials $p(x)$ of degree ≤ 3 choose $\{1, x, x^2, x^3\}$ as a basis and let D be the derivative mapping:$Dp(x) = \frac{d}{dx}p(x)$. Represent D by a matrix. 6

Section C

Q 6 (a) In a plane, T maps [x,y] into $[x+y, x-y]$. Find the matrix representing T with respect to the basis $\{(1,-1),(1,1)\}$. 6

(b) Show that the polynomials $(1-t)^3$, $(1-t)^2$, $(1-t)$ and 1 generate the space of polynomials of degree≤ 3. 4

Q 7 (a) Define a Skew-Hermitian matrix. Show that the diagonal elements of a skew-Hermitian matrix are either zero or purely imaginary. 6

(b) Show that the determinant of an orthogonal matrix is either $+1$or -1. 4