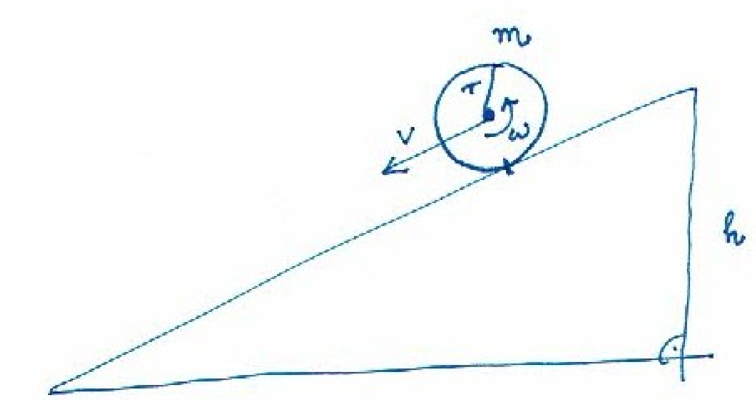
Inclined plane

Rolling down rotationally symmetrical bodies

Betting roles

Moments of inertia

Conservation of energy with potential energy, kinetic energy and rotational energy



How do different rotationally symmetrical bodies roll down an inclined plane:

Solid cylinder, hollow cylinder, solid ball, hollow ball

Note: Interestingly, for the result of the betting roll, it doesn't matter what mass or radius the individual bodies have!

Examples:

Full cylinder piece of a wooden broomstick, sanded beverage can

Hollow cylinder piece of a hollow metal rod, empty beverage can, toilet paper roll

Solid steel ball, sand filled ball, marble

Hollow ball ball

Task:

Find out in which order the bodies arrive below!

Solution:

1st full ball

2. Full cylinder

3. Hollow ball

4. Hollow cylinder

Task:

Find the formulas for the moments of inertia of the four bodies and use the energy conservation theorem to compare the speeds of the bodies.

Solution:

The moments of inertia are all of shape

I = q m r2

where m is the mass and r is the radius of the body.

(Again: m and r may be different for each body!)

and

q has a different value for each body:

Solid ball 2/5

Solid cylinder 1/2

Hollow ball 2/3

Hollow cylinder 1

At the end of the inclined plane, the body's initial potential energy has completely transformed into kinetic and rotational energy:

$$E_{pot} = E_{kin} + E_{Rot}$$

The rotational energy is given by the expression

$$E_{Rot} = \frac{1}{2} J \omega^2$$

analogous to kinetic energy

$$E_{kin} = \frac{1}{2} m v^2$$

where I denotes the so-called moment of inertia of the rotating body.

This is an implicit definition of I.

Intermediate task:

Calculate the moment of inertia of a hollow cylinder!

Solution:

$$E_{Rot} = \frac{1}{2} J \omega^2$$

$$\omega = \frac{v}{r}$$

$$E_{Rot} = \frac{1}{2} J \frac{v^2}{r^2} = \frac{1}{2} \frac{J}{r^2} \cdot v^2$$

omega and v are linked together

$$v = \omega \cdot r$$

This is the implicit definition of omega, the so-called angular frequency.

The rotational energy is also the kinetic energy of the mass when it rotates around the axis of rotation:

$$E_{kin} = \frac{1}{2} m v^2$$

$$E_{Rot} = E_{kin}$$

$$\frac{J}{r^2} = m$$

$$\boxed{J = m r^2}$$

End of the intermediate task

We now skillfully transform the equation so that instead of I the expression q = I / (m r2) is created and omega is replaced by v:

$$E_{pot} = E_{kin} + E_{Rot}$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} J \omega^2$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} m \frac{J}{m} \cdot \frac{v^2}{r^2}$$

$$v = \omega \cdot r$$

$$\omega = \frac{v}{r}$$

$$mgh = \frac{1}{2} m v^2 + \frac{1}{2} m v^2 \left(\frac{J}{mr^2} \right)$$

$$\cancel{m} g h = \left(1 + \frac{J}{mr^2}\right) \frac{1}{2} \cancel{m} v^2$$

$$2gh = \left(1 + \frac{J}{mr^2}\right) \cdot v^2$$

$$2gh = (1 + q) \cdot v^2$$

The equation states that the smaller the value of q, the greater the final velocity v. Since this also applies to any instantaneous speed, it applies to the entire movement along the inclined plane.

The equation is actually independent of m and r of the body.

The sequence to be determined in the experiment is now theoretically proven!

Note: I am not aware of an insightful solution about a force approach ....

Category	
Title	**Betting roles, Rolling down rotationally symmetrical bodies, Inclined Plane**
Physical subject matter	**Mechanics, Conservation of Energy, Moments of inertia**
Learning level	5
Preparation difficulty	3
Price per set/€	
Attractiveness	3
Standart-exotic	3
Instructions set-up	yes
Instructions execution	yes