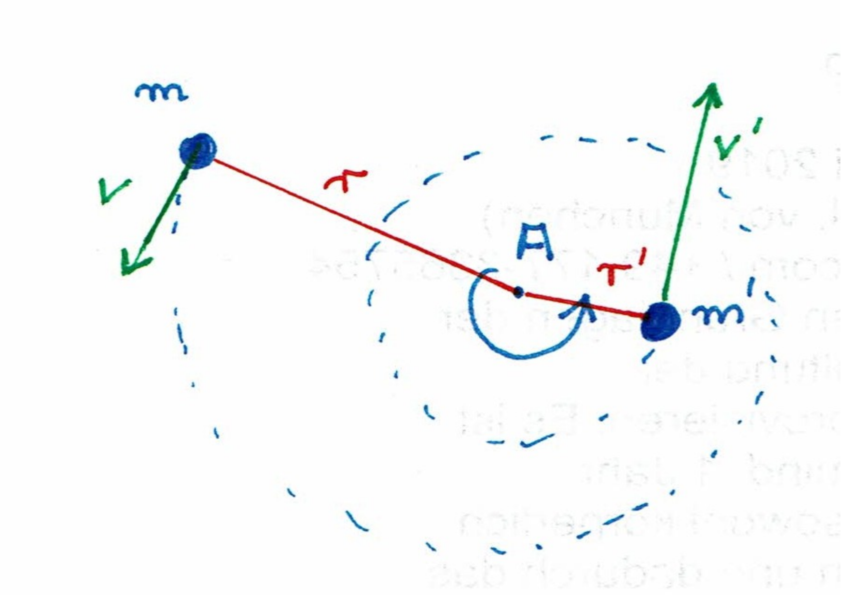
Vortex as a result of conservation of angular momentum

Task 1:
Explain why a water particle m in a water vortex rotates the faster (i.e. higher rotational velocity v) the closer it is to the axis of rotation! This happens when it is sucked into the vortex.

Solution:
The law of conservation of angular momentum must apply to the water particle m:



$$m \cdot v \cdot r = m \cdot v' \cdot r'$$

This means that the product of mass, speed and turning radius must remain constant!

$$v(r) \sim \frac{1}{r}$$

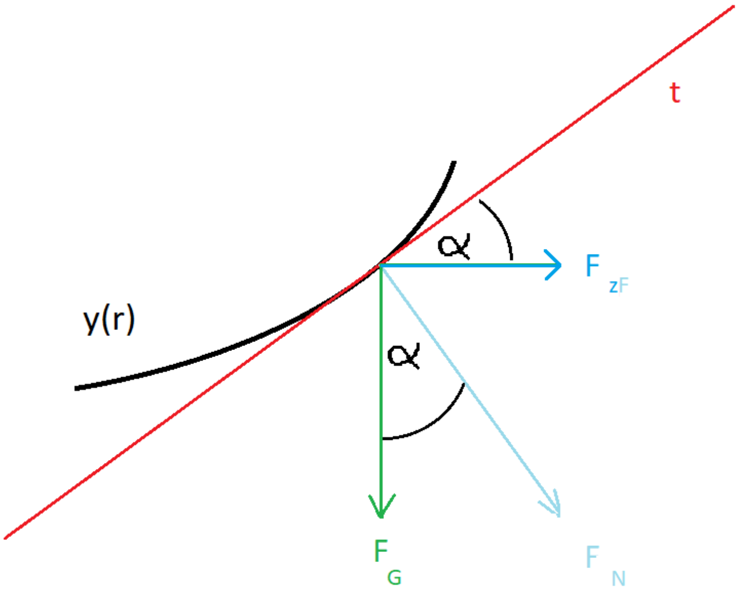
So its speed v is always indirectly proportional to the radius:

(1)

Task 2:
Show: The surface forms a rotational hyperboloid!

Tip:
The inclination of the water surface (angle α) results from the sum of the forces of the centrifugal force and the weight of the volume piece. (Forces in the rotating reference system)

Solution:



$$\tan\alpha = \frac{F_{zF}}{F_G} \quad (2)$$

If ω is the rotational frequency, the following applies

$$\tan\alpha = \frac{m \cdot \omega^2 \cdot r}{m \cdot g}$$

$$\tan\alpha \sim \omega^2(r) \cdot r$$

use of

$$\omega(r) = \frac{v(r)}{r}$$

results

$$\tan\alpha \sim \frac{v(r)^2}{r}$$

because of (1)

$$v(r) \sim \frac{1}{r}$$

applies

$$\tan\alpha \sim \frac{1}{r^3}$$

The slope of the graph is also the derivative of the surface function y (r) after r:

$$y'(r) = \tan \alpha$$

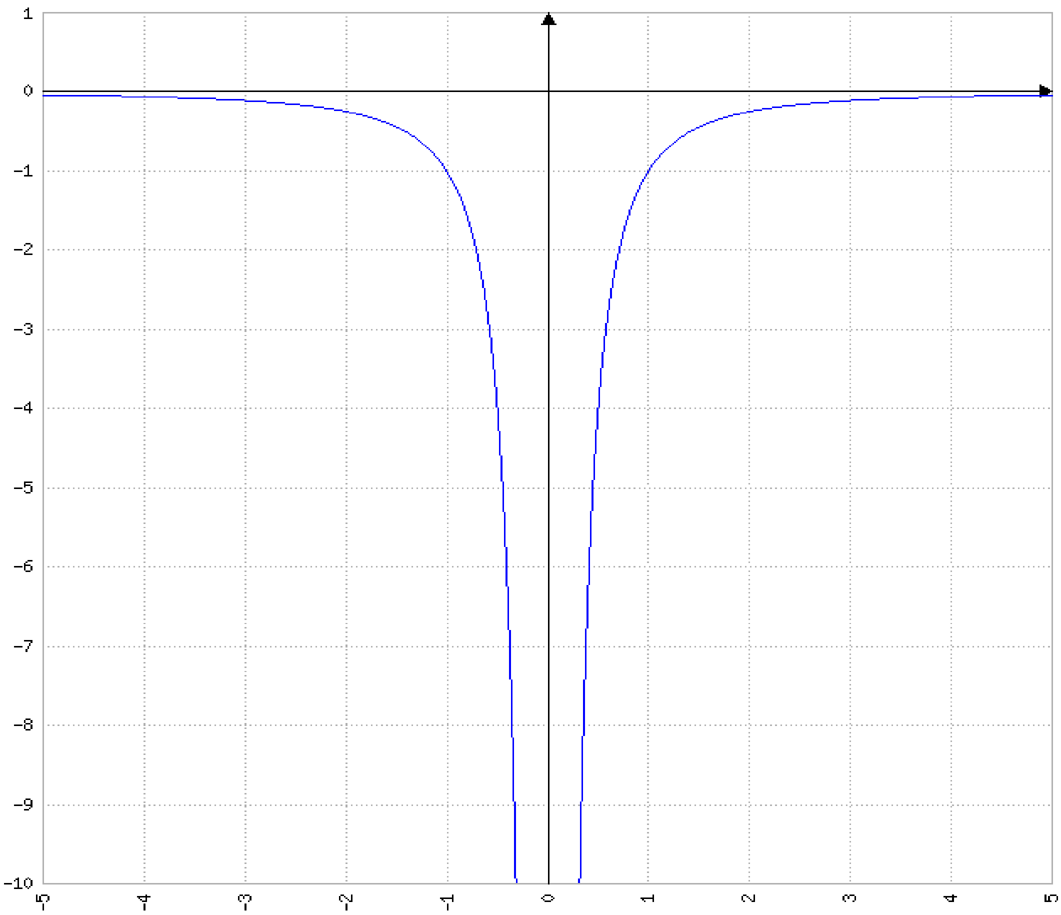
so

$$y'(r) \sim \frac{1}{r^3}$$

and thus

$$y(r) \sim \frac{1}{r^2}$$

Result:
The surface of the vertebra is a rotational hyperboloid.



Outlook:
A vertebra of water or vortex can be seen as a system analogous to a black hole. E.g. there is also an “event radius” there, a distance from which an external water wave is inevitably drawn into the vortex.
Analogous to Hawking's theory about black holes, the incoming wave extracts energy from the vortex. It is therefore weakened.