MHD Waves and Instabilities in an Anisotropic Plasma Medium Consisting of Ultrarelativistic and Nonrelativistic Components in Relative Motion

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Abstract

The propagation of waves and instabilities in a plasma medium consisting of concatenation of two magnetohydrodynamic (MHD) fluids by including the effect of relative motion at relativistic speed among the two fluid components. One of the fluid is an ultrarelativistic $[\mathbf{kT_{\parallel,\perp} \gg mc^2}]$ plasma moving at relativistic speed with respect to the background non-relativistic anisotropic plasma. The ultrarelativistic fluid has anisotropic temperature described by double adiabatic equations. The pressure components of the anisotropic background plasma are given by generalized polytropic laws. The linearized analysis is carried out and dispersion relation is derived using normal mode technique. The dispersion relation is discussed both analytically as well as numerically for parameters which simulate cosmic rays and the background inter-stellar medium. An additional mode of propagation, the suprathermal mode, is the only mode which transports energy in the direction perpendicular to the ambient magnetic field. This mode is affected by the presence of relative motion between the components. For large enough value of the relative speed between the components its phase speed vanishes in the direction perpendicular to the magnetic field. In this case the system does not allow transport of energy transverse to the magnetic field. Situations where this model supports instabilities are also discussed.

INTRODUCTION

Many astrophysical objects move outward from their source of origin and may pervade regions having a qualitatively different plasma which in many cases may be non-relativistic. The interaction between these two plasma systems may influence collective phenomena, in particular propagation of hydromagnetic waves in this region. It is then desirable to describe the medium as a concatenated system of two infinitely conducting MHD fluids coupled together by the ambient magnetic field. The nature of interaction between the two fluids is determined by the scale length of the small-scale irregularities in the large scale ambient magnetic field. A simple model consisting of a thermal interstellar plasma mixed with a suprathermal cosmic ray plasma was pioneered by Parker[1]. Recently, Kumar and Singh[2] investigated the effect of relative motion among the two components of the concatenated plasma medium in a nonrelativistic domain. In the present study the concatenated system simulates the cosmic ray fluid by an ultrarelativistic anisotropic plasma streaming with relativistic speed relative to the background inter-stellar medium which is considered to be a nonrelativistic anisotropic plasma.

BASIC EQUATIONS

The magnetic field constrains the two fluids to move together in a direction transverse to the field but it cannot prevent them from moving independently in the direction along the field. Assuming an equilibrium flow velocity of $\mathbf{V_0\hat{z}}$ for fluid 1, the basic equations of the problem are obtained by linearization of the set of basic equations of the concatenated model given by Kalra and Kumar[3]. Under the assumption of perfect conductivity, Ohms law together with Faraday's law gives the following linearized equation for the perturbed magnetic field(denoted by **b**):

$$\frac{\partial \mathbf{b}}{\partial \mathbf{t}} = \nabla \times (\mathbf{v_1} \times \mathbf{B_0} + \mathbf{V_0} \times \mathbf{b}) = \nabla \times (\mathbf{v_2} \times \mathbf{B_0}), \tag{1}$$

It is evident from Eq.(2) that the corresponding perpendicular components of the velocities of the two fluid are related as:

$$\mathbf{v_{1x} = v_{2x} + V_0\frac{b_x}{B_0}}, \quad \mathbf{v_{1y} = v_{2y} + V_0\frac{b_y}{B_0}}. \tag{2}$$

Here the pressure of the plasma components is given by

$$\mathbf{p_{1,2} = (p_{\parallel_{1,2}} - p_{\perp_{1,2}})nn + p_{\perp_{1,2}}I},$$

where $\mathbf{p_{\parallel_{1,2}}}, \mathbf{p_{\perp_{1,2}}}$ denote the pressure parallel and perpendicular to the direction of magnetic field, respectively. **n** denotes a unit vector along the direction of the magnetic field and **I** represents the unit second order tensor. For the closure of the relativistic gyrotropic MHD equations, the above equations are supplemented by the equations of state (EOS).

- EOS for Fluid1: Generalization of CGL laws for the ultrarelativistic case

$$\mathbf{\frac{p_{\perp_1}}{p^0_{\perp_1}} = \frac{\rho_1}{\rho^0_1} + \frac{b_z}{2B_0}},$$

$$\mathbf{\frac{p_{\parallel_1}}{p^0_{\parallel_1}} = 2\frac{\rho_1}{\rho^0_1} - \frac{b_z}{B_0}}.$$

- EOS for Fluid2: The double polytropic law

$$\mathbf{\frac{p_{\parallel_2}}{p^0_{\parallel_2}} = \gamma_{\parallel_2}\frac{\rho_2}{\rho^0_2} - (\gamma_{\parallel_2} - 1)(\frac{b_z}{B_0})},$$

$$\mathbf{\frac{p_{\parallel_2}}{p^0_{\parallel_2}} = \gamma_{\parallel_2}\frac{\rho_2}{\rho^0_2} - (\gamma_{\parallel_2} - 1)(\frac{b_z}{B_0})},$$

where $\gamma_{\parallel_2}$ and $\gamma_{\perp_2}$ are the polytropic indices of fluid 2.

DISPERSION RELATIONS

Any arbitrary perturbation of the system can be assumed to have the form of a plane wave and can be expressed in the form $\propto \mathbf{\exp\{i(\omega t - k \cdot r)\}}$. Carrying out the usual normal mode analysis, one gets the following dispersion relations

$$\mathbf{(\rho^0_1 R^*_1 + \rho^0_2)\frac{\omega^2}{k^2} - 2V_0\frac{\omega}{k}\rho^0_1 Q^*_1 \cos\theta - \{\frac{B^2_0}{4\pi} + \sum_{s=1}^{2}(p^0_{\perp_s} - p^0_{\parallel_s}) - V^2_0\rho^0_1 Q^*_1\}\cos^2\theta = 0}, \tag{3}$$

$$\begin{aligned}\mathbf{\{\rho^0_1 Q^*_1(\frac{\omega}{k} - V_0\cos\theta)^2 - 2p^0_{\parallel_1}\gamma^2_0(\frac{V_0\omega}{c^2\,k} - \cos\theta)^2\}} &\times \Big[\mathbf{(\rho^0_2\frac{\omega^2}{k^2} - \gamma_{\parallel_2}p^0_{\parallel_2}\cos^2\theta)} \\ \mathbf{(\rho^0_1 R^*_1 + \rho^0_2)\tfrac{\omega^2}{k^2} - \tfrac{B^2_0}{4\pi} - 2V_0\rho^0_1 Q^*_1\tfrac{\omega}{k}\cos\theta} &\mathbf{- \cos^2\theta\{\textstyle\sum_{s=1}^{2}(p^0_{\perp_s} - p^0_{\parallel_s})} \\ \mathbf{-V^2_0\rho^0_1 Q^*_1\} - \sin^2\theta(\frac{3}{2}p^0_{\perp_1} + \gamma_{\perp_2}p^0_{\perp_2})} &\mathbf{- \sin^2\theta\cos^2\theta(p^0_{\perp_2})^2}\Big] \\ \mathbf{-\gamma^2_0(\rho^0_2\frac{\omega^2}{k^2} - \gamma_{\parallel_2}p^0_{\parallel_2}\cos^2\theta)(\frac{V_0\omega}{c^2\,k}} &\mathbf{- \cos\theta)^2(p^0_{\perp_1})^2\sin^2\theta = 0.}\end{aligned} \tag{4}$$

where

$$\mathbf{R^\star_1 = \gamma^2_0 + \frac{1}{\rho^0_1 c^2}[(1+2\gamma^2_0)p^0_{\perp_1} - (1-2\gamma^2_0)p^0_{\parallel_1} + \frac{B^2_0}{4\pi}]}, \quad \mathbf{Q^\star_1 = \gamma^2_0[1 + \frac{2}{\rho^0_1 c^2}(p^0_{\perp_1} + p^0_{\parallel_1})]}.$$

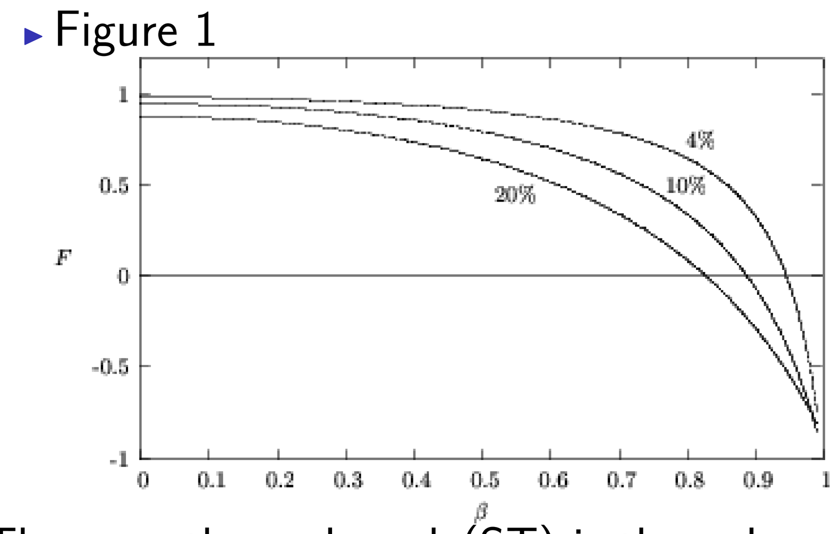
The general dispersion relation of the present problem is eighth order in ω indicating the existence of eight wave modes (four forward and four backward propagating independent disturbances). This is the outcome of the two-population MHD model.

NUMERICAL COMPUTATIONS

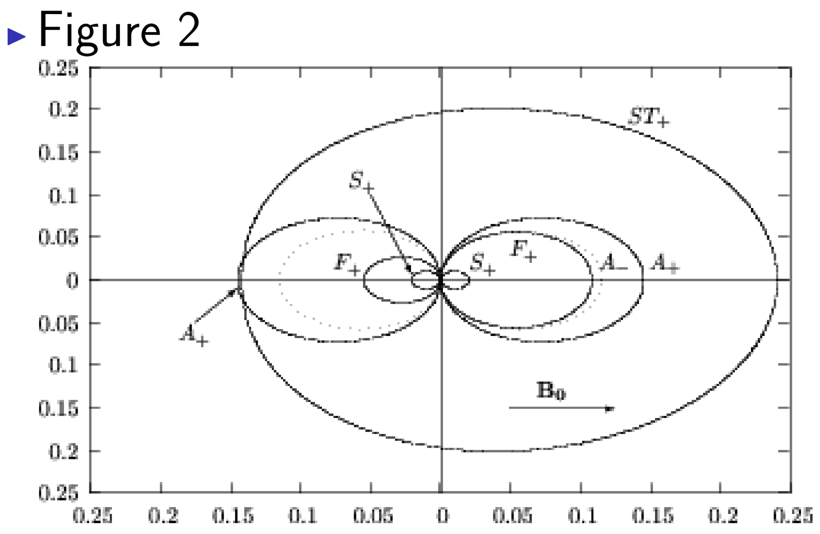
The situation in space can simulated to be a concatenation of two anisotropic MHD fluids composed of a cosmic ray fluid (component 1: ultrarelativistic) and background inter-stellar plasma (component 2: nonrelativistic).To investigate such a situation, we adopt the relevant astrophysical parameters[1, 4]. The system does not support propagation of any MHD modes and the system is *firehose* unstable for those parameters which satisfy $\mathbf{F = 1 + \sum_{s=1}^{2}(s^2_{\perp_s} - s^2_{\parallel_s}) - \beta^2\alpha^2 Q^*_1(1 - \Lambda) < 0}$, where $\mathbf{\beta = V_0/c}$, $\mathbf{s^2_{\perp_s,\parallel_s} = 4\pi p^0_{\perp_s,\parallel_s}/B^2_0}$, $\mathbf{\alpha^2 = 1/(1+d)M^2_A = c^2/V^2_A}$.

$\mathbf{V_A(= B_0/\sqrt{4\pi\rho^0_1})}$ is the classical Alfvén speed in fluid 1 and $\mathbf{d(= \rho^0_2/\rho^0_1)}$ is the ratio of densities of fluid 2 to fluid 1. Figure 1 shows the plot of **F** versus β, the relative speed between the components normalized to speed of light for different density ratios **d** of the two MHD components comprising the system. Figure 2 depicts the polar plot of phase speeds of the four modes (slow, fast, Alfven and Suprathermal)of wave propagation in the system.

- Figure 1

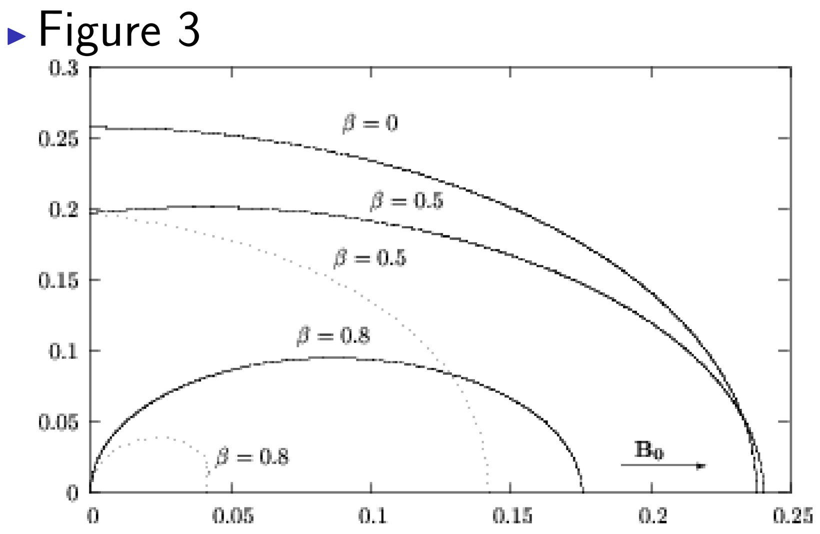


- Figure 2



The suprathermal mode(ST) is the only mode which propagates in a direction perpendicular to the magnetic field. Its propagation characteristics are examined in Fig. 3 for different values of relative speed. For large enough value of relative speed of motion among the components, the phase speed of both the forward and backward propagating suprathermal mode vanishes in a direction perpendicular to the magnetic field.

- Figure 3



In this case there is no transport of energy in a direction transverse to the magnetic field. This feature of ST is a result of relative motion at relativistic speed among the components of the two-population plasma system.

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