Chapter 1

# Introduction to Discrete-Time Control Systems

## 1-1 INTRODUCTION

The use of digital or discrete technology to maintain conditions in operating systems as close as possible to desired values despite changes in the operating environment. Traditionally, control systems have utilized analog components, that is, controllers which generate time-continuous outputs (volts, pressure, and so forth) to manipulate process inputs and which operate on continuous signals from instrumentation measuring process variables (position, temperature, and so forth). In the 1970s, the use of discrete or logical control elements, such as fluidic components, and the use of programmable logic controllers to automate machining, manufacturing, and production facilities became widespread. In parallel with these developments has been the accelerating use of digital computers in industrial and commercial applications areas, both for logic-level control and for replacing analog control systems. The development of inexpensive mini- and microcomputers with arithmetic and logical capability orders of magnitude beyond that obtainable with analog and discrete digital control elements has resulted in the rapid substitution of conventional control systems by digital computer-based ones. With the introduction of computer-based control systems into major consumer products areas (such as automobiles and video and audio electronics), it is clear that the digital computer will be widely used to control objects ranging from small, personal appliances and games up to large, commercial manufacturing and production facilities.

**Types of signals:**

**Continuous Time Signal**: A signal is called **continuous-time** if it is defined at every time t. A system is a continuous-time system if it takes a continuous-time input signal, and outputs a continuous-time output signal.

Continuous time signal is **defined over a continuous range of time**.

**Analog Signal**: Continuous range of time & continuous range of values in amplitude Analog signal is a special case of continuous-time signal

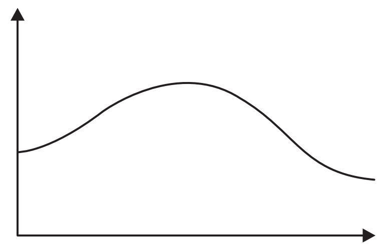


Fig. 1.1(a): Continuous-time analog signal

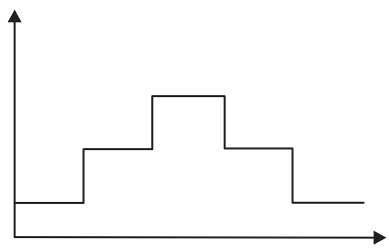


Fig. 1.1(b): Continuous-time quantized signal

Discrete time signals are further divided in two parts namely sampled data signal and digital signal. Sampled data signal are pulse amplitude modulated signals and are obtained by some means of sampling an analog signal. A pulse amplitude modulated signals is presented in the form of a pulse train with signal information carried by the amplitudes of the pulses.Digital signals are a discrete time signal with quantized amplitude such a signal can be represented by a sequence of numbers. It is a signal quantized both in amplitude and in time.

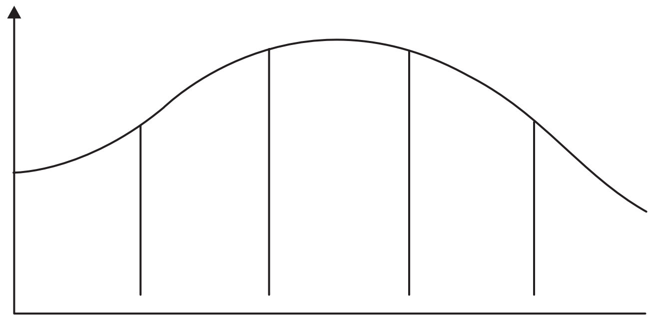


Fig. 1.2(a): Sampled data signal

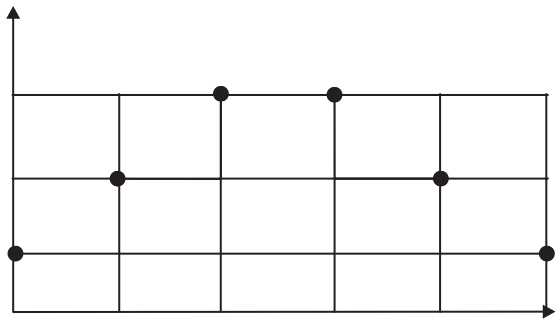


Fig. 1.2(b): Digital data signal

## 1.2 SAMPLED DATA SYSTEMS

A control system where the continuous-time plant is controlled with a digital device is a sampled-data system. Under periodic sampling, the sampled-data system is time-varying but also periodic, and thus it may be modeled by a simplified discrete-time system obtained by discretizing the plant. However, this discrete model does not capture the inter sample behavior of the real system, which may be critical in a number of applications.

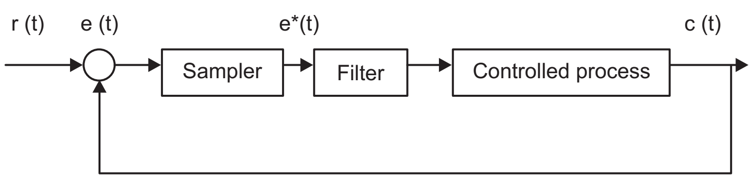



Fig. 1.3: Closed loop sampled data control system.

The Fig. 1.3 shows the basic elements of a typical closed loop control system with sampled data. The sampler represents a device that outputs a pulse train. No information is transmitted between two consecutive pulses. The magnitude of the pulses at the sampling instant represents the values of the input signal $(e(t))$ at the sampling instants. The filter is used for the purpose of smoothing.

## 1.3 DIGITAL CONTROL SYSTEMS

The rapid increase in the use of digital controller in the controlled system is due to its achievement in the optimum performance. Digital control system provides optimal performance in the form of maximum productivity, maximum profit, minimum cost or minimum energy use etc. The application of computer control has made possible the intelligent motion in industrial robots, the optimization of the fuel economy in automobiles and refinement in the operations of house hold appliances and machines such as microwave ovens, washing machine, Air-conditioning. Decision making capability and flexibility in the control programs are major advantages of digital control system. The current trend towards rather then analog control system is mainly due do the availability of low cost digital computers and the advantages found in working with digital signals rather then continuous time signals.

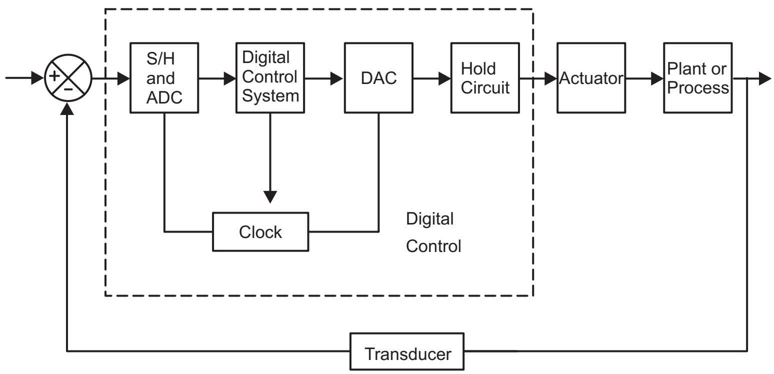



Fig. 1.4: Block diagram of digital control systems.

The controller operation is performed or controlled by the clock. In such a DCS points of the systems pass signals of varying amplitude either in continuous time or discrete time or in numerical code.

**Sample and Hold (S/H):** It is the circuit that receives an analog input signal and holds these signals at a constant value for a specified period of time. Usually the signal is electrical but it may be optical or mechanical.

**ADC:** ADC also called an encoder is a device that converts an analog signal into a digital signal, usually a numerically coded signal in binary form. Such a converter is need as an interface between an analog component and the digital component. Basically ADC involves sampling, quantizing and encoding.

**Digital Computer:** The digital computer processes the sequences of numbers by mean of an algorithm and produces new sequences of numbers.

**DAC:** DAC also called a decoder is a device that converts a digital signal (Numerically coded data) into an analog signal. It acts as an the interfacing device between the digital component and an analog component. The real time clock in the computer synchronizes the events. The output of the hold circuit which is continuous time signal is fed the plant either directly or through the actuator which controls the dynamics of the system (i.e. it smoothes the slope of the signal).

**Plant or Process:** A plant is a physical object to be controlled. The examples are a furnace, chemical reactors and a set of machine parts functioning together to perform a particular operations such as servo system etc.

**Transducer/Sensor:** A transducer is a device that converts an input signal into an output signal of another form such as device that converts a temperature into a voltage output (thermistor or thermocouple) and an optical signal into voltage (phototransistor).

**Discrete Time Control System:** Discrete time control system is control system in which one or more variable can change only at discrete instants of time. These instants which are denoted by KT or $t_k\left(k = 0,\ 1,\ 2\ ....\right)$, specify the times at which some physical measurements are performed. The time interval between two discrete instants is taken to be sufficiently short that the data for the time between them can be approximated by simple interpolation.

## 1.4 DATA ACQUISITION, CONVERSION AND DISTRIBUTION

## Data Acquisition System

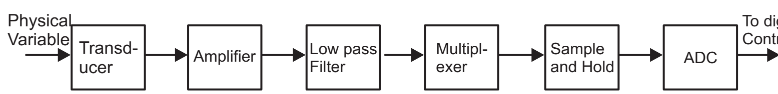



Fig. 1.5: Basic block diagram of data acquision system.

Fig. 1.5 shows the diagram of data acquisition system. The basic parameters are explained below:

**1. Physical Variable:** The input to the system is a physical variable such as position, velocity, acceleration, temperature, pressure etc.

**2. Transducer Amplifier and Low Pass Filter:** The physical variables (which are generally in non-electrical form) is first converted into an electrical signal (a voltage or a current signal) by a suitable transducer. Amplifier then amplifies the voltage output of the transducer (i.e. the signal have rises to the necessary level). The LPF follows the amplifier which attenuates the high frequency signal components such as noise signals which are random in nature. The o/p of LPF is an analog signal. The signal is then fed to an analog multiplexer.

**3. Analog Multiplexer:** It is a device that performs the function of time sharing and ADC among many analog channels. It is a multiple switch (usually an electronic switch) that switches sequentially among many analog input channels in some prescribed fashion. The no of channels may be 4,8,16.

**4. Sample and Hold Circuit:** A simpler in a digital system converts an analog signal into a train of amplitude modulated pulses. The hold circuit holds the value of the sampled pulse signal over a specified period of time. It is necessary in the AD converter to produce a number that accurately represents the i/p signal at the sampling instant.

**5. ADC:** The output of sample and hold is then fed to the AD converter. The o/p of the converter is the signal in digital form which is fed to the digital controller. In this way data acquisition system is held**.**

**Data Distribution:**

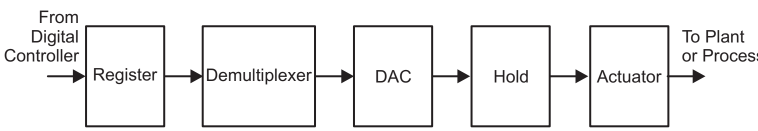



Fig. 1.6: Basic block diagram of data distribution system.

**1. Register:** The o/p of digital controller is then stored for a certain period of time in a memory device called register.

**2. Demultiplexer:** The demultiplexer, which is synchronized with the i/p sampling signal, separates the composite o/p signal which is in the from of digital data from the digital controller into the original channels. Each channel is connected to DAC to produce the o/p analog signal for that channel.

**3. DAC:** At the o/p of the digital controller, the digital must be converted to an analog signal by the process called D/A conversion. For the full range of digital i/p, there are 2n different analog values, including zero.

**4. Hold:** The sampling operation produces an amplitude modulated pulse signal. The function of hold operation is to reconstruct the analog signal that has been transmitted as a train of pulse samples. The purpose of hold operation is to fill the spaces between the sampling periods and thus roughly reconstruct the original analog input signal which is then fed to the actuator which smoothens the slope of signal.

**5. Plant or Process:** A plant is a physical object to be controlled. The examples are a furnace, chemical reactors and a set of machine parts functioning together to perform a particular operations such as servo system etc.

**Data Conversion Process:**

**Signal Sampling, Quantizing and Encoding** Signal sampling is the first step of transmission of analog signal over digital signal.

Sampling is the process of conversion of continuous time analog signal into discrete time analog signal. The discrete signal obtained after sampling is called sampled signal.

**Sampling Theorem:** It states "Analog signal can be reproduced from an appropriate set of its samples taken at some fixed intervals of time." This theorem has made possible to transmit only samples of analog signal by changing or encoding this samples into block of code words suitable for digital control systems.

If $fs$ = sampling frequency

$fx$ = maximum frequency component of the i/p signal,

then the distortion less recovery of the signal $fs \geq 2fx$

If the signal $x(t)$ to be sampled is band limited, then the sampled signal can be represented as:

$$xs(t) = x(t) \times g(t)$$

where, $g(t)$ is the sampling function (rectangular pulse train) which be represented as shown below.

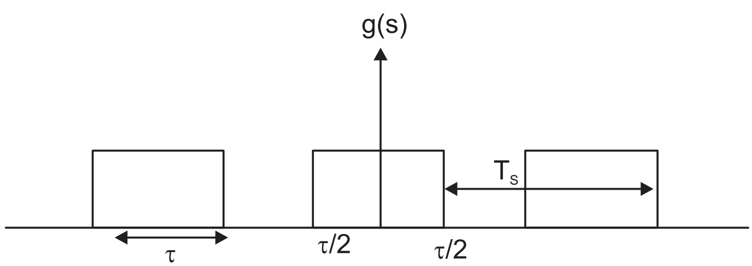



Fig. 1.7: Rectangular pulse train

where,

$T_s$ = Sapling period.

$\tau$ = duration of sampling pulse = pulse width.

Sampler can be implemented as:

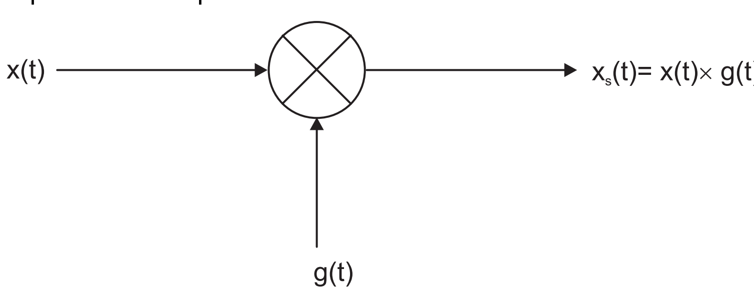



Fig. 1.8: Implementation of sampler

**Proof of sampling theorem:** The gate function $g\,(t)$ can be expressed in terms of Fourier Series as

$$g(t) = c_o + \sum_{n=1}^{\infty} 2c_n \cos(nw_s t)$$

where, $co = \tau / Ts = \tau fs$

$$\begin{aligned} Cn &= fs\ \tau\ \sin c\left[nfs\ \tau\ \right] \\ &= co\ \sin c\left[nfs\tau\right] \\ \omega s &= 2\ \pi fs \end{aligned}$$

The signal $\mathrm{xs}(\mathrm{t})$ can be expressed as

$$\begin{aligned} xs &= x(t) \times g(t) \\ &= x(t)\left[c_o + \sum_{n=1}^{\infty} 2c_n \cos(nw_s t)\right] \\ &= co\ x(t) + 2c1x(t)\ \cos\ \omega nt + 2c2x(t)\cos 2 \end{aligned}$$

$$\omega st + .............+ 2cnx(t)\cos n\omega st + .............$$

The Fourier transform of above series as

$$\begin{aligned} xs(f) &= co(f) + 2c1x(f - fs) + 2c2x(f - 2fs) + ............. \\ &\quad + 2cn\ x(f + nfs) + ............. \end{aligned}$$

The above series can be graphically represented as:

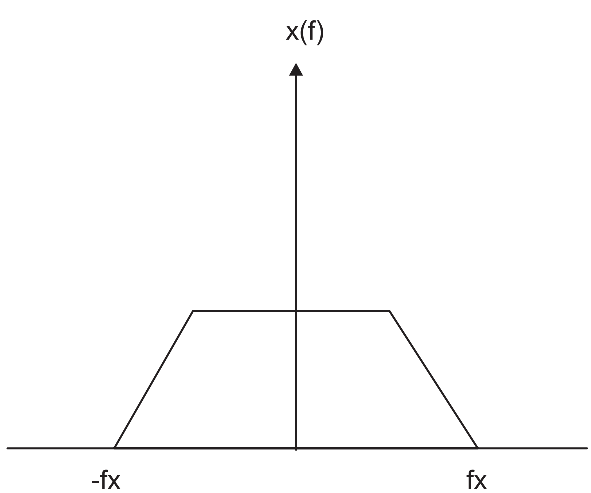



Fig. 1.9: Message spectrum

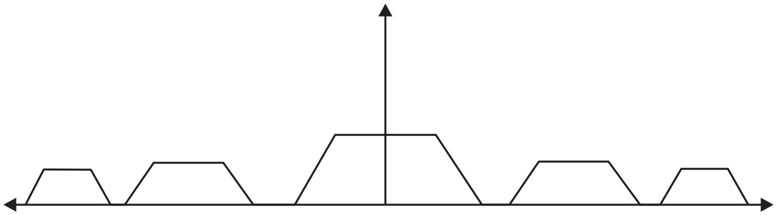


Fig. 1.10: Spectrum of the sampler

It is clear from Fig. 1.10 that the spectrum of the sampled signal contains the spectrum of the sampled signal contains the spectrum of the original message signal.It is evident that for distortion less recovery of original message signal, from the spectrum of the sampled signal, the following condition should be met, $fs \geq fx$. In this case the original message signal spectra can be recovered by passing the sampled signal through low pass filter with bandwidth equaling to $\pm fx$.. Distortion will occur while recovering the message spectrum if $fs \leq fx$. The distortion in the above case is caused by the overlapping of side bands and message spectra. as shown in Fig. 1.11.

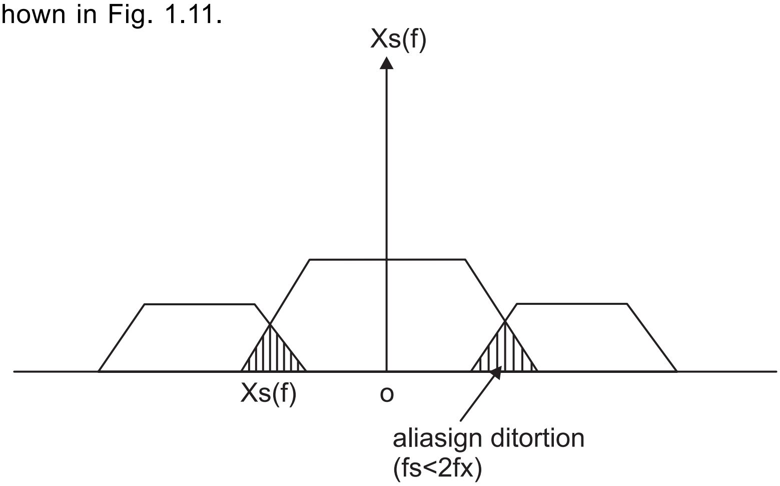



Fig. 1.11(a): Overlapping of side bands and message spectra.

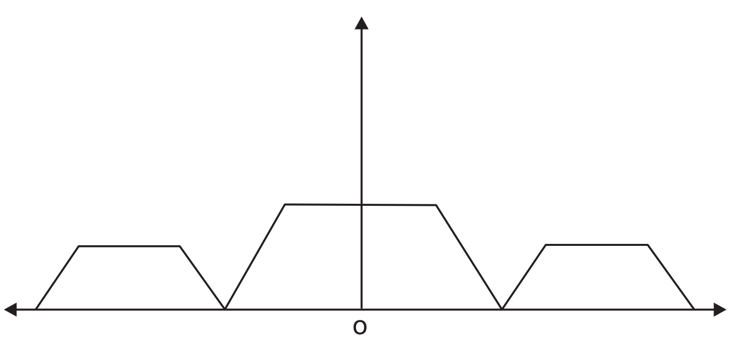



Fig. 1.11(b):

The minimum sampling rate: $fs\ \min = 2fs$ is called Nyquists's sampling rate for distortion less recovery of one message spectrum. The minimum interval of the sampling for a real signal is

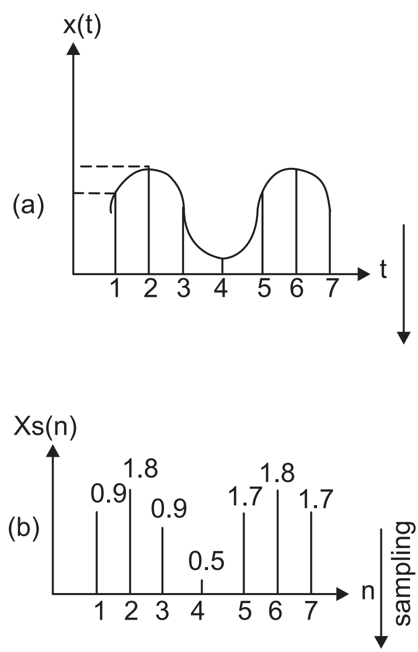
$$Ts\ \min\ =\ 1/2fx(\min)$$

where,

$fx(\min)\ =$ maximum frequency in the message spectrum.

## Quantizing and Quantization Error

Quantization is the process of representing the analog sample values by a finite set of levels. The sampling process coverts a continuous time signal to a discrete time signal with amplitude and that can take any values from zero to maximum level and the quantization process converts continuous amplitude samples to a finite set of discrete amplitude values. The output states of each quantized sample are then described by a numerical code. The process of representing a sampled value by a numerical code is called encoding. Thus encoding is a process of assigning a digital word to each discrete state. In uniform quantization it is assumed that the range of input sample is $-x_{\max}$ to $x_{\max}$ and the number of quantization level, known as Q-level, $N = 2^n$. Where, $n\ =$ is the number of bits per source sample is the number of bits per source sample, then the step size $\Delta$ on length of the Q Level is assumed to be,



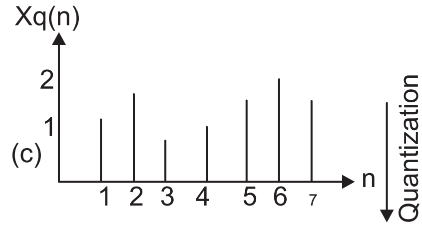



Fig. 1.12(a): Continuous- continuous value signal

1.12(b): Discrete time- continuous value signal

1.12(c): Discrete time- discrete value signal.

$$\Delta = 2x_{\max} / N = 2x_{\max} / 2^{n-1}$$
$$\Delta = x_{\max} / 2^{n-1}$$

The step size $'\Delta'$ is also called 'quantum'.

**Quantization Error:**
Since the number of bits in the digital word is finite, ie the digital output can assume only a finite number of levels and therefore analog number must be round off to the nearest digital level. Hence any ADC involves quantization error. It is evident that the maximum Q error could be only $\Delta / 2$. In uniform quantization the steps size $\Delta$ is constant for the entire dynamic range of the input discrete signal level. Q error depends on the fineness of the quantization level and can be made as small as desire by making the quantization level smaller or by increasing the number of bits '$n$'. In practice there is a maximum for n and so there is always some error due to quantization.

The uncertainty present in the quantization process results quantization noise.

**Signal to Quantization Noise Ratio (SQNR):** It is evident that the Q-error (i.e. qe) lies Between $-\Delta / 2$ to $\Delta / 2$ in random manner, the average power of Q-noise is therefore given by,

$$P_q = \frac{1}{\Delta} \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} q_e^{\,2} dq_e$$

$$= P_q = \frac{\Delta^2}{12} \qquad \text{..........}(1.1)$$

It is seen from Eq. (1.1) that Q-noise is dependent on step-size '$\Delta$' only. Reducing the Step-size or increasing the no of representation level, we can reduce $pq$ and hence Q-error.

From Eq. (1)

$P_q = \Delta^2 / 12$

But,

$$\Delta = x_{\max} / 2^{n-1}$$

$$P_q = (x_{\max} / 2^{n-1})/12 = x_{\max}{}^2 / 3 * 4^n \qquad \text{............}(1.2)$$

Assume that the average signal power is $x^2, SQNR$, for uniform quantization will be,

$SQNR =$ average power of signal / average power of noise.

$$QSNR = \hat{x}^2 \times 3 \times 4^n \qquad \text{.............}(1.3)$$

where, $\hat{x}^2 = \overline{x}^2 / x^2 =$ Normalized signal power..

Again, Eq. (1.3) can be reproduced in $dB$, as

$$\begin{aligned}(QSNR)_{dB} &= P_x(dB) + 10\log_{10}{}^3 + 10\log_{10}(4)^n \\ &= P_x(dB) + 4.8 + 10n\log_{10}\end{aligned}$$

where,

$P_x(dB) = 10(\log_{10}(\hat{x}^2)dB$

Thus,

$$(QSNR)\ dB = Px + 4.8 + 6n \qquad \text{...............}(1.4)$$

Also,

$$SQNR\ (dB) = Px + 4.8 + 20\ \log 10^N \qquad \text{............}(1.5)$$

$[\text{since, } N = 2n]$

And for $N >> 1$

$SQNR = 20\ \log 10N$ .......... Which is approximated $SQNR$ for uniform quantization in terms of level of quantization "$N$".