5. 

## Problem 11.50E (HRW)

Two particles, each with mass $m$, are fastened to each other, and to a rotation axis at $O$, by two thin rods, each with length $l$ and mass $M$ as shown in the figure. The combination rotates around the rotation axis with angular velocity $\omega$. Obtain algebraic expressions for (a) the rotational inertia of the combination about $O$ and $(\mathrm{b})$ the kinetic energy of rotation about $O$.


## Solution:

(a)

As mass of each rod is $m$ and their combined length is $2 l$, the moment of inertia of the rods with respect to rotation axis at $O$ is $\frac{1}{3} \times 2 M \times(2 l)^{2}=\frac{8}{3} M l^{2}$.

The moment of inertia of the two balls about $O$ is $m l^{2}+m(2 l)^{2}=5 m l^{2}$.

Therefore, the algebraic expression for the rotational inertia of the combination about $O$ is $I=\frac{8}{3} M l^{2}+5 m l^{2}$.
(b)

And, the kinetic energy of rotation about $O$ is

$$
K E_{\text {rot }}=\frac{1}{2} \times I \omega^{2}=\frac{1}{2}\left(\frac{8}{3} M+5 m\right) l^{2} \omega^{2} .
$$

